Input Impedance of Horns with Crooks: Measurements and Modelling

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Abstract:

Horns are conical brass instruments; when played without the use of valves they can only sound the frequencies at which a standing wave can be established. In order to increase the number of notes that can be played with such an instrument, crooks are used. They are bits of tubing which can be inserted in the instrument to extend the total length of the bore and therefore set up a different set of standing waves. The different types of crooking systems – master crook and couplers, terminal and inventionshorn – are discussed from a practical point of view, in terms of intonation and when coupled to lips. We use the BIAS system to measure the input impedance of a number of horns. Some horns are then modelled using the BIAS software and a FDTD model. This enables us to discuss the effect of crooks on the playing of horns and factors influencing the choice of crook for an instrument.

Declaration

I do hereby declare that this dissertation was composed by myself and that the work described within is my own, except where explicitly stated otherwise.

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Table of Contents

1. Introduction

1.1 Horns

The Horn as we know it today is a conical brass instrument with valves. In its earlier stages of development though, horns had no valves and could therefore only play a limited range of notes. In order to modify the length of the tubing and extend the number of notes available to the player, crooks were used. We describe these early horns without valves as natural horns.

1.1.1 Natural Horns

Natural horns are brass instruments which precede the modern horn; they consist of a mouthpiece, a mouthpipe, coiled tubing and a large flaring bell but no valves. The coiled tubing is in fact 'part cylindrical, part conical' (Humphries 2000, 27). As natural horns have no valves to modify the length and thus the sounding pitch of the instrument, their sounding pitches are limited to the natural modes of resonance of the instrument. These are the frequencies at which standing waves will occur in the air column when the lips of the player excite it. Although the natural modes of resonance often approximate the harmonic series, it is important to remember that they are not equivalent. This affects the harmonicity of the instrument; we find for example that the 7th, 11th and 13th harmonics are out of tune. The notes between the 7th and the 14th resonance approximate a diatonic scale while higher resonances are approximately chromatic. Composers have used this knowledge to compose horn parts which would otherwise have been reduced to simple chordal sequences.

The lack of valves means that the instrument's scope for modulations is limited; it becomes difficult to integrate the instrument in an orchestra. An expensive solution would be for players to own various instruments in different keys; an impractical solution as composers explored further tonal regions. As Humphries points out, even if a player owned instruments in different keys, they could not be finely tuned to the orchestra (Humphries 2000, 28). The solution for natural horn players is the crook system. The types of crook systems we will discuss are 'master crook and couplers', 'terminal crook' and 'inventionshorn'.

1.1.2 Master Crook and Couplers

It is generally acknowledged that horns with crooks were first built by Leichnamschneider as early as 1700 (Humphries 2000, 28. Baines 1976, 156. Hiebert 1997, 104); trumpets with crooks had already appeared during the 17th century (Carse 1939, 215). The crook system consists of a tapered master crook which accepts a mouthpiece. This can then be attached to any number of couplers which achieve the desired length of tubing. This system is practical because it requires a little amount of extra equipment to be carried with the player; a large number of keys can be crooked through different combinations. The inconvenience of this system is that every combination will modify the size and shape of the instrument so that the player is almost confronted with a new instrument each time. The shortest coupling position results in having the instrument very close to the face. Longer couplings, say five couplers and a master crook, result in the instrument being played far from the body. Such a construction can also be rather wobbly, which is a crucial flaw as the player requires a precise embouchure.

The bore profile of horns (a combination of conical and cylindrical) adds a difficulty for the instrument maker. Careful attention needs to be paid to preserving the correct bore profile of the instrument while inserting straight or coiled bits of tubing. Baines suggests the horn must be divided 'at a point where the tube reached approximately trumpet width', the larger part 'be refashioned to form a hoop smaller than before, or with three coils reduced to two, and with a socket for the crooks placed at the top of the hoop (Baines 1976, 156-157).

1.1.3 Inventionshorn

The inventionshorn represents a compromise to solve the issues thrown up by the couplers. First manufactured in 1750 by Johann Werner (Humphries 2000, 28), the crooks are inserted in the middle of the instrument, where one of the hoops has been cut and bent upwards in parallel fashion to receive crooks. While this solution means that the instrument is always held at the same distance of the face, it is problematic because of the lack of space in the instrument. As the crooks must fit within the coils, longer crooks cannot be used. Popular keys for this sort of instrument lie between D and G. A similar invention is Raoux's 1760 cor solo which has body crooks in G, F, E, Eb and D (Humphries 2000, 29). As we will discuss later, the inventionshorn also suffers from the fact that the bore profile is changed significantly with every change of crooking.

1.1.4 Terminal Crook

A third solution is the terminal crook system, where each key's required tube length is achieved by its own crook. The musician carries a crook for each key in his case and changes them as required. The advantage of this system is that the instrument remains at an equal length from the player in each key as the terminal crook coils. The obvious problem though is the amount of supplementary metal that needs to be transported by the player. Reportedly some musicians owned up to 16 terminal crooks!

1.1.5 Valves

The invention of the valve in the early 19th century was a major turning point for brass instruments (Humphries 2000, 32). Valves lengthen the distance travelled by waves when depressed; it should then be possible to play all notes without the need for crooks. But players and manufacturers alike were reticent to adopt them. We must realise that each crook gives a different feel to the instrument as their acoustic properties change. Humphries describes the subjective difference between the Bb alto to the Bb basso crook, as the difference between driving a sports car and a lorry (Humphries 2000, 31). Therefore players deplored the loss of the different playing characteristics of horns with different crooks. Even though the valve was patented in 1818, valved horns were only completely adopted in 1903 (Humphries 2000, 16). As a result, a large number of valved horns were built with terminal crooks. It was felt that this compromise allowed to expand the number of notes available to the player while preserving the different sonorities that crooks provide. This is the reason why a number of valved horns were used in this project; it would be erroneous to solely look at natural horns. All valved instruments will be measured without depressed valves; discussion on the effect of the valve section of the tubing on horns will have to omitted here.

It is worth mentioning the omnitonic horn as a further crook system. It is a natural horn with a full set of crooks which can be brought into play by a mechanism on the instrument. This mechanism is not equivalent to valves as the key cannot be changed during play and it requires the right hand which is also used for hand muting. The omnitonic horn will not be measured as the only example in the Edinburgh collection is not in a measurable condition.

1.1.6 Hand Muting and Lipping

Hand muting is a horn technique where the right hand is inserted into the bell in order to modify the sound of the instrument. It allows to reach pitches otherwise inaccessible on a natural horn, as well as correcting intonation. A full stop causes the pitch to drop down to a semitone above the previous harmonic. This effect can be further helped by lipping the note down. Thereby the player adapts his embouchure to move away from the peak of resonance (Norman 2013, xvii-xviii). This technique is critical for natural horns; for example the 11th resonance lies between F and F# - both notes often used by composers – so the player needs to lip up or down to play the required note. The ease with which the instrument reacts to lipping partly determines its quality as it is easier to play the required notes in tune. (Norman 2013, 67). These two factors are critical to musical performance on horns. However, a detailed discussion of hand muting would be far beyond the scope of this project. The effect of lipping will be considered when discussing modelling results.

1.2 Input Impedance and Harmonicity

Horns consist of a mouthpiece, a mouthpipe, a main bore and a flaring bell. The horn is a resonator driven by a lip reed; the buzzing of the lips acts as an outward-striking reed (Adachi and Sato 1996, 1200). The player's lips effectively act as valve through which the air flow enters the instrument. The pressure fluctuations they create in the mouthpiece produce 'over pressure impulse' which travels down the instrument (Widholm 2008, 72). Most of this wave is reflected at the bell while a very small part is radiated out of the bell. The reflected wave combines with the incoming wave to produce a standing wave. This happens if the time it takes the wave to do a 'round-trip' of the instrument is the same time it takes the lips to open and close. It will also produce a standing wave at integer multiples of this period (Braden 2006, 3). So when a brass instrument is sounded at a certain frequency, it sets in vibration this frequency and integer multiples of the given frequency.

1.2.1 The input impedance of an instrument is the ratio of the pressure in the mouthpiece to the output produced by the instrument. It is denoted as Z and defined as

$$Z = \frac{p}{vS} \qquad [1]$$

where,

p = Acoustic Pressure, Pa

v = Velocity, m/s

S = Surface Area, m^2

The input impedance is measured in Acoustic Ohms. In fact, pressure is measured in Pa and flow is measured m^3/s so the acoustic ohm is measured in $\frac{Pa}{m^3/s}$, which is also $\frac{Pa.s}{m^3}$.

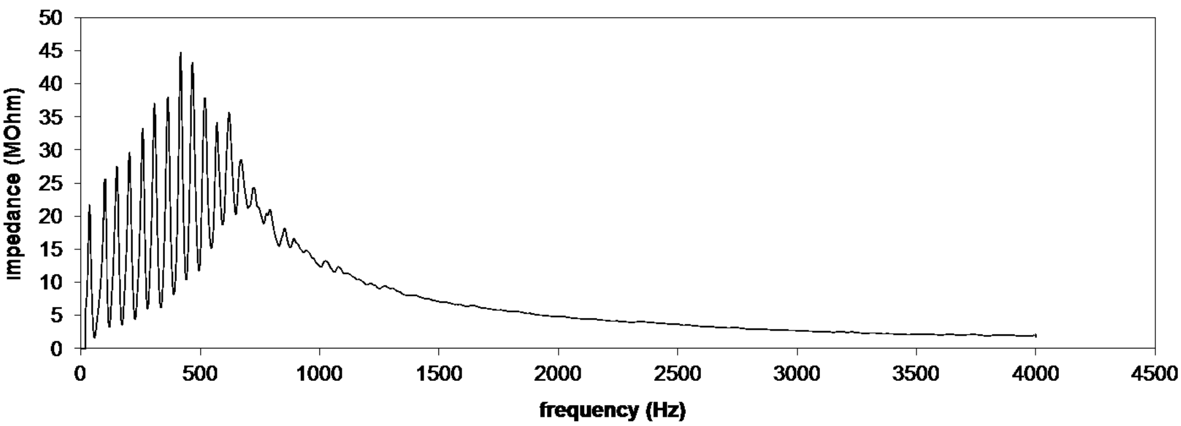


Fig. 1: Input impedance of 4092 crooked in G.

The input impedance can tell us at which frequencies the instrument's natural resonances lie, as the air flow will be highest at those frequencies. Playing a note at a frequency with low input impedance will be easier than a frequency with high input impedance. The latter will be difficult to sound and will have low strength. If input impedance peaks are multiples of a fundamental frequency, they will couple in an oscillatory regime. The more resonances can be coupled in this regime, the richer and stable the played note will be. It is then desirable for the input impedance of an instrument to align so that such regimes can develop. This can be of particular interest for the fundamental note of the instrument, as it is most often very flat. Yet, it can still be sounded as its harmonics vibrate in a regime corresponding to the fundamental (Benade 1973, 33).

We must also take into account the effects of frictional and thermal losses in the instrument; these increase with frequency. The energy leaking out of the bell also increases at higher frequencies. It is then no surprise to find that the peaks in the impedance curve are strongly reduced at high frequencies. No substantial energy is reflected from the bell at 1500 Hz and above (Benade 1973, 29).

The Q-value of a peak is the bandwidth of the peak. A higher value of Q (meaning a narrower peak) narrows the target frequencies for the player; the embouchure needs to be more precise. Larger peaks give more freedom for lipping as neighbouring frequencies have good strength (Norman 2013, 80).

What does this mean for the player? The ease of playing and quality of a note is determined by the height of the input impedance peak, the harmonicity of the input impedance and the Q-value of the peaks. It is also worth noting that the dynamic of a note affects it sound as the coupling of frequencies is intensified with volume. At a low level of playing, only the lipped frequency will vibrate; as the level increases the second mode, the third mode, etc. are brought into play. This effect is described by the Worman theorem (Benade 1973, 71). Instruments are said to sound 'bright' or 'dull', which is correlated to whether higher harmonics couple with the played frequency.

1.2.2 Harmonicity

The harmonicity of an instrument describes the relationship between the natural resonances as it shows where each resonance lies in relation to an arbitrary frequency. The *Equivalent Fundamental Pitch* (EFP) can be plotted by calculating for each peak frequency f_i the fundamental frequency of which f_i is the i^{th} harmonic. EFP is calculated relative to an arbitrary frequency F; in our case F $= \frac{f_4}{4}$. This is because players tune their instruments to the fourth resonance (Braden 2006, 88). EFP is given by

$$EFP = \frac{1200}{\log(2)} \log\left(\frac{f_i}{iF}\right) \qquad [2]$$

The distance of the peaks from the alignment of the harmonic series is given in cents. EFP is plotted on the horizontal axis rather than the usual vertical axis for clarity.

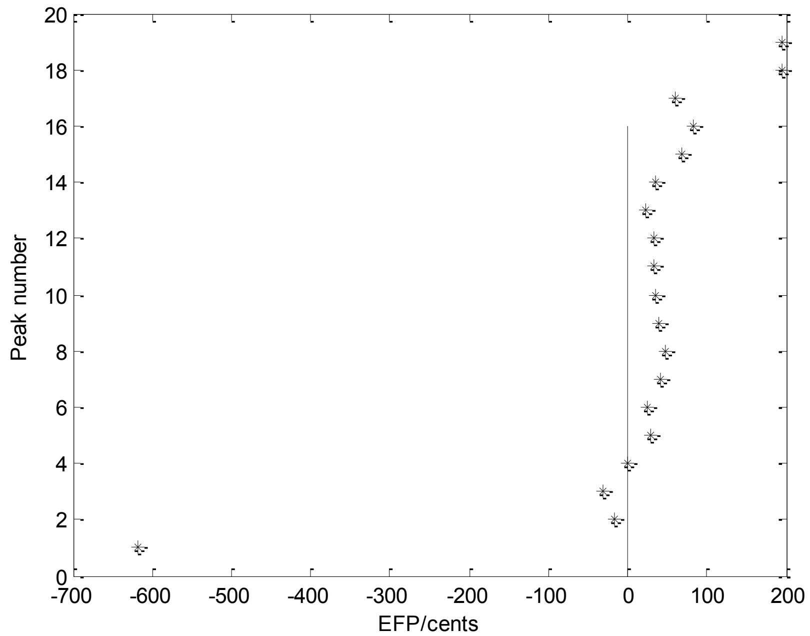


Fig. 2: EFP of the instrument 4092 crooked in G. The first resonance is very flat which is usual in brass instruments, it is therefore disregarded for this purpose.

1.3 Mouthpiece and bore shape

1.3.1 Mouthpiece

We will now discuss some of the factors that influence an instrument's natural modes of resonance, explaining the fact that they will most often not correspond to the harmonic series. We will first consider the example of a conical tube. This would be a very crude approximation of a horn, which certainly has a conical section but also a mouthpiece, mouthpipe, cylindrical sections (i.e. the valve section) and rapidly flaring bell. A conical tube has natural modes of resonance which are indeed equal to the harmonic series. In order to accommodate a mouthpiece though, about 10% of the beginning section of the tube is 'cut off' and replaced by a mouthpiece of similar total volume (Campbell and Greated 1987, 334). We find that the modes of resonance go sharp when the apex is removed. When a mouthpiece with the same volume as the removed apex is used, the first four are almost harmonics before going sharp. When a mouthpiece larger than the volume of the apex is used, 'the impedance peaks of the tube are flattened by an amount which increases with frequency up to, and beyond, the mouthpiece resonance frequency' (Campbell and Greated 1987, 335). Additionally, resonances tend to go sharp above the mouthpiece's resonance frequency (Pyle 1975, 1315).

The mouthpiece has a resonance frequency which can also influence the input impedance curve of an instrument. In fact, the air in the mouthpiece acts a Helmholtz resonator (Cardwell 1970 in Campbell and Greated 1987, 330), the resonance frequency is therefore amplified; which can be observed on the input impedance curve. This property can determine the sound quality decisively; if the mouthpiece increases the 'impedance multiplication' of high resonances, the instrument will sound more bright and brilliant (Campbell and Greated 1987, 34). So the mouthpiece can both amplify and shift input impedance peaks.

1.3.2 Bore shape

While the right choice of mouthpiece can address the intonation of higher modes of resonance, the lower ones remain very flat due to the cylindrical sections of tubing in the instrument. A flaring bell is a major factor in solving this problem, as to bring the modes of resonance closer to the harmonic series. This is due to the flaring shape, which increases the wave speed by an amount depending on the curvature of the wall (Morse 1948, 265-288 in Campbell and Greated 1987, 345). At lower frequencies, the increase in speed is consequently higher. At the reflection point, this speed becomes infinitely large and the wave is reflected. As this reflection point moves further inside the bell, the length of the wave is shortened and the mode of resonance becomes sharper. Unfortunately, this effect is lowest on the first mode which remains resolutely flat. Gorgerat states that the more the bell shape flares, the more chance there is that instrument plays out of tune (Gorgerat 1955, 63). This intuitive statement is in fact due to this phenomenon. The bell shape also affects the cut-off frequency, above which the vibrations are strongly attenuated. This is counteracted by the fact that higher frequencies radiate straight out of the bell; therefore the playing of very high notes is possible. But they will only be sounded weakly as they cannot be 'slotted' in an input impedance peak; what we would hear is almost just the lip frequency. (Chick et al. 2010).

Pyle describes the shape of a horn's bell with the Bessel Horn equation, where $0.8 \leq \alpha \leq 1.2$ describes a typical French Horn. In fact, he equates $\alpha = 0.8$ to Viennese Horns and $\alpha = 0.9 – 1.0$ to German instruments and $\alpha = 1.2$ to larger-throated American instruments (Pyle 1975, 1312).

1.3.3 Equivalent Cone Length

The *Equivalent Cone Length* for each mode of resonance frequency, which length of cone would produce that harmonic at the described frequency. For an nth resonance at frequency f_n, the Equivalent Cone Length is

$$L_e = nc/2f_n$$

where c is the speed of sound (Pyle 1975, 1309). To approximate the modes of resonance to the harmonic series produced by a cone, an instrument should produce notes with all the same Equivalent Cone Length. This is a helpful concept in describing the intonation of instruments, as it describes similar trends as the Equivalent Fundamental Pitch.

1.3.4 Crooks

Crooks are used to extend the instrument tubing to the required length. As previously discussed, one of the difficulties in building a good crook is that of maintaining an approximately conical bore shape. The crooks measured were not found to be cylindrical. The intonation will be affected by this further deviation from the ideal conical shape. Usually the bore shape will be adjusted to work best with medium length bores (Baines 1976, 163); Eb, E, F and G crooks are effectively the most popular choices of crooks.

2. Methodology

2.1 Instruments

Thirteen instruments from University collections have been used, they are kept at the University Library, the Reid Concert Hall Museum of Instruments and St Cecilia's Hall Museum of Instruments. The instruments will be referred to by their EUCHMI catalogue number that can be found in Tab. 1. Only the crookings measured are included, more detailed information can be found in the EUCHMI catalogue.

Catalogue Number	Catalogue Name	Maker, Origin and Date	Type of Crooking	Crookings Measured
203	Orchestral Hand Horn	W. Sandbach London (1810-1830)	Master Crook and Couplers	C, D, Eb, E, F, G, Ab, A (2 crookings)
2888	Orchestral Hand Horn	Anon. Prob. England (late 18th cen.)	Master Crook and Couplers	C alto
3296	Orchestral Hand Horn	J.C. Hofmaster London (ca. 1760)	Master Crook and Couplers	A
6144	Cor solo	Raoux Paris (1823)	Inventionshorn	D, Eb, E, F, G
533	Valved Horn	Boosey and Co. London (1879)	3-valve, Inventionshorn	Eb, E, F, sharp F, G
531	Orchestral Hand Horn	Kretzschmann Strasbourg (1830)	Terminal	B basso, C, D, Eb, E, F, G, A, Bb
4668	Orchestral Hand Horn	Courtois neveu Paris (ca. 1840)	Terminal	C, D, Eb, E, F, G, A, B
204	Orchestral Hand Horn	J.G. Kersten Dresden (ca. 1830)	2-valve, Terminal	B basso, Db, D, Eb, E, F, G, Ab, A, Bb alto
208	Orchestral Horn	Rudall Carte	3-valve,	C, D, E, G, Ab

		London (ca. 1893)	Terminal	
1874	Orchestral Horn	C. Mahillon Brussels or London (1878 – ca. 1900)	2-valve or 3-valve (detachable), Terminal	E (2 crookings), F (3 crookings), G (2 crookings), A, Bb
3198	Cor d'harmonie	Raoux / Millereau Paris (ca. 1880)	3-valve, Terminal	F, G, A
4671	Orchestral Horn	Gautrot Paris (ca. 1875)	3-valve, Terminal	C, D, Eb, E, F, G, Ab, A
4092	French Horn	R.J. Ward Liverpool (early 20th cen.)	3-valve, Terminal	D, Eb, F, G, Ab

Tab. 1: Instruments used with Catalogue Number, Catalogue Name, Maker, Origin and Date, Type of crooking and crookings measured.

2.2 Measuring

2.2.1 BIAS

The Brass Instrument Analysis System (BIAS) was used to measure the input impedance of the horns. BIAS is a commercially available capillary-based method consisting of a measuring head which can be connected to a computer which operates the BIAS software. The measuring head contains a speaker and two microphones which are situated at either end of a high-impedance capillary. A sinusoidal chirp from 0 – 4096 Hz is sent through the speaker in a period of two seconds while the reference and recording microphones record the pressure. These correspond to the incoming pressure and outgoing air flow. Although the version of BIAS software used only measures from 0 – 4096 Hz rather than up to 20,000 Hz, this range is largely sufficient in the context of musical instruments (Sharp et al. 2010, 820). A mouthpiece is inserted in the measuring head; the same horn mouthpiece was used for all the measurements (AM 1266 lent by Arnold Myers). Players would probably choose different mouthpieces for different instruments, but the use of multiple mouthpieces would be beyond the scope of this project.

Consistency in measurements can be difficult, particularly as background noise influences the measurements. When measuring in the Reid Collection of Instruments, it was not always possible to measure in quietness due to the presence of visitors and builders. Each crook was measured three times to ensure optimal results. The BIAS software assumes a room temperature of 21 C°. At the time of measuring the temperature in the Library collection was 22 C°, 20 C° in the Reid Hall and of 21 C° at St Cecilia's Hall. Temperature influences the absolute pitch of an instrument but not its harmonicity, therefore no adjustments were made to the input measurements to account for the small temperature differences.

The constitution of the player's lips affects the intonation; the further the lips protrude into the mouthpiece, the smaller the effective volume of the cup and the sharper the pitch (Widholm 2008, 6). It is possible to simulate the lip position in the mouthpieces for different brass instrument by resting the measured mouthpiece on rubber mats of different shapes. Again, the same rubber mat was used for all measurements for consistency between measurements.

BIAS' compact size and speed of measurement and processing makes it a convenient method to measure instruments located in collections. The instruments measured are located at the Reid Concert Hall, St Cecilia's Hall and the University of Edinburgh Main Library. Additionally, the fragile condition of certain objects make it imperative to measure them in the collection.

Not all instruments could be measured satisfactorily due to the shape of BIAS' measuring head. Certain horns with master crook and couplers did not fit the measuring head and gave useless results. This is because the master crook is coiled so that when the lips are applied to the mouthpiece, the player's head is very close to the instrument. When trying to fit the instrument onto the flat surface of the measuring head, the mouthpiece could not be fully inserted into the crook (as illustrated in Fig. 3). The affected instruments are 2887, 2888, 3296 and 3297 (Fig. 4; their measurements were therefore discarded. 203 was measured but suffered from the same problem to a lesser degree.

Fig. 3: Instrument 2888 with a master crook and an additional master crook and coupler. The red line shows that it is not possible to correctly insert the instrument into the flat measuring head. (http://www.mimo-db.eu/UEDIN/2888)

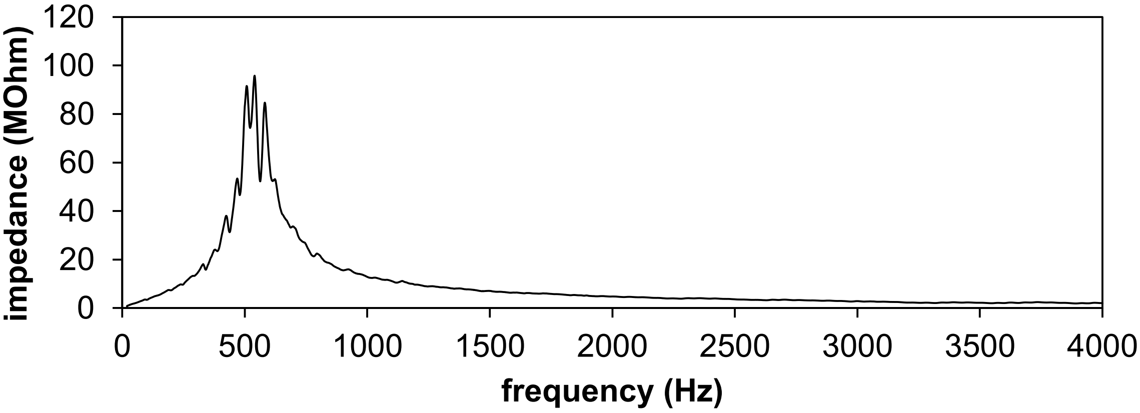


Fig. 4: Input impedance of 3296 crooked in G.

2.2.2 Evaluating BIAS data

The input impedance plots were exported from BIAS as .via files and then processed in Matlab to extract their peaks. Unfortunately the code does not distinguish between strong and 'weak' peaks; it is then usual to find over 25 peaks although it is clear that these are very weak in reality. The EFP is then calculated from the input impedance peaks.

2.2.3 Methods for measuring Input Impedance

The BIAS system is a capillary-based method. Similar methods have often been used to calculate the input impedance of instruments. Benade (Benade 1973, 27) describes a setup where a driver from a horn loudspeaker provides stimulus to an instrument with two microphones recording the pressure and the air flow. A second method by Merhaut (Benade 1973, 28) is presented where the acoustic stimulus vibrates a diaphragm which pumps air into the mouthpiece and controls the frequency of the incoming pressure. Backus (Backus 1976, 470) describes a similar system which uses a driver unit and one microphone in the mouthpiece. Pratt (Pratt et al. 1977, 239) proposes a method where the measurements are taken at the mouthpipe rather than the mouthpiece so that the input impedance of the instrument can also be measured without the mouthpiece. Other examples of work with capillary-based systems includes Caussé et al. 1984, Kemp et al. 2007, and Sharp et al. 2011. BIAS has been used in some recent research (Norman 2013, Chick et al. 2010, and Kausel 2003). Capillary-based methods remain popular due to their simplicity in setup, the resulting measurements require no processing to remove the coupling between the instrument and the apparatus, and the range in which they work is perfect for musical instruments (Sharp et al 2011, 820).

2.3 Modelling

The second part of the project uses modelling to compare and discuss the experimental results. The BIAS software models the input impedance and impulse response of an instrument. The second model was provided by Jonathan Kemp and is based on Stefan Bilbao's work on the finite difference time domain simulation of brass instruments (Bilbao 2013). It couples a brass instrument to a lip model, thus modelling the playing behaviour rather than the input impedance only.

2.3.1 Measuring the instruments

The instruments to be modelled are 533 (Inventionshorn), 1874, (Terminal), 4092 (Terminal), 4671 (Terminal) and 203 (Master Crook and Couplers). Both models require the diameter of the tube and its axial distance along the instrument. The measurements of the mouthpiece (AM 1266) were provided by Arnold Myers and the measurements of 203 were provided by Lisa Norman. The diameter of the tubing was measured with Vernier calipers; 0.8 mm were then deducted from the reading as we assumed the wall thickness to be 0.4 mm. A tape measure was used to determine the axial distance of each reading. As tubing tends to become increasingly elliptic with time, it was sometimes necessary to measure the diameter at different angles and average the readings. Another difficulty were dents in tubing as they make it harder to accurately measure the diameter of the instrument at that point. The aim was to record the original diameter at that place rather than the diameter including the dent (this will be discussed later on).

The BIAS software can also measure the pulse response of the instrument. A short and strong impulse is sent through the tube and the reflections of this impulse are measured, which can then determine the acoustical length of the instrument. BIAS calculates the pulse response from the impedance data (BIAS). Measuring the instrument with different crookings also gives the length of the individual crooks. This method was used to double check the measurements made by hand, to ensure no major errors had occurred.

2.3.2 BIAS

The BIAS software provides the possibility to model the impedance of instruments numerically. As the software is built with instrument makers in mind, a physical model gives the chance to predict the behaviour of an instrument without needing to build it.

This model is based on the Transmission Line Analogy which simulates One Dimensional Wave Analogy (Widholm 2008, 141). The instrument is simulated as a series of conical and cylindrical slices. The sound pressure difference is modelled by the drop in voltage in a transmission line and the viscothermal losses are modelled by resistances. Every transmission line has a characteristic impedance so that

$$Z = V_{in}(t)/I_{in}(t) \qquad [3]$$

where V is the alternating current voltage and I is the current. This is analogous to the input impedance of an instrument as it is the ratio between incoming pressure and outgoing flow.

The impedance of each segment is

$$A_i(f) = \begin{matrix} a_{i11}(f) & a_{i12}(f) \\ a_{i21(f)} & a_{i22}(f) \end{matrix} \qquad [4]$$

The Pressure (p) and Flow (u) are linked so that

$$\begin{matrix} p_i(f) \\ u_i(f) \end{matrix} = A_i(f) \begin{matrix} p_{i+1}(f) \\ u_{i+1}(f) \end{matrix} \qquad [5]$$

And

$$Z_i(f) = \frac{p_i(f)}{u_i(f)} \qquad [6]$$

The overall transmission matrix is then the product of all the single matrices,

$$A(f) = \prod A_i\,(f) \qquad [7]$$

(Widholm 2008, 141)

The series of the transmission line segments is ended by a termination impedance which models the partial reflection of waves at the bell. The model uses the equilibrium gas density, the radian frequency, the shear viscosity, the speed of sound, the planar, cross-sectional area at the centre, the spherical area at the input end of the conical element, the radius of the input spherical sector, the radius of the output spherical sector and the distance between the two spheres. Full details can be found in Kausel 2013.

2.3.3 FDTD Simulation

The Finite Difference Time Domain model for brass instruments models lossy linear wave propagation taking into account the thermodynamic losses at the walls. This is then coupled to a lip model as described by Adachi and Sato which supposes that a lip is a single mass which can move in two dimensions and as a forced damped harmonic oscillator (Adachi and Sato 1996). The model is described in detail in Kemp et al. 2013.

The coupling of a lip model to FTDT model adds an important parameter to our previous discussion. The input impedance tells us about the response of the instrument under stable and repeatable conditions, as is a sinusoidal chirp played by a speaker. But a human player cannot play a sinusoidal chirp at constant amplitude. Neither does the input impedance describe the coupling a lip to the instrument. By modelling this response we hope to understand more about the playing qualities of the instrument. This can be particularly interesting for historical instruments as these, as most cannot or should not be played anymore.

The lip model simulates slur, we will use an upward slur of 150 Hz; the result simulates the instrument played with a lip slur. The result we will examine is the plot of the playing frequency plotted against time.

The model has been used in a number of research projects (Norman 2013, Newton et al 2014) which look at the effect of lipping in brass instruments. A brass player is asked to play a note and then lip upwards or downwards until the instrument jumps to the next resonance. In both cases the measured and modelled results have been in broad agreement which showed that the player's lipping sensation can be deduced from the model. In a comparison between two 18^{th} century horns, the horn which allows for easier pitch bending while playing also allows for a larger pitch bend when simulated (Norman 2013, 538). Thus the model seems to be modelling the pitch bending behaviour well enough to give relevant indications on the playing qualities of an instrument.

2.3.4 Parameters

The **BIAS** model requires the input of diameter and axial distance while temperature and damping are assumed to be constant unless specified. Temperature is assumed to be 21 C˚ and damping a factor of 1. A change in room temperature changes the absolute pitch of the horn but not its relative intonation (harmonicity). However, the BIAS manual suggests that the temperature inside the instrument goes from 36 C˚ (as breathed out) to 21 C˚ (room temperature) as it cools down in the tubing. It might therefore be of interest to specify these temperature gradients in the model for a more accurate representation. An increase of the damping factor can account for wall roughness so dents in the tubing can be modelled without the need of modifying the diameter of the bore. The temperature and damping factors were left as default in most simulations. A large dent in 4092 was simulated by augmenting the damping factor to 2 and then 4 for the length of the dent.

The **FDTD** model also needs defining a temperature which was kept at 20 C° throughout. The mass and the quality of the factor of the lip were kept the same as in Kemp et al. 2013. These parameters were used to simulate a trumpet, it was thought that the lip conditions would be very similar with a horn. The interval of frequencies between 300 Hz and 450 Hz is also appropriate for the modelling of a horn as it lies roughly in the area of the harmonic which approximates a diatonic scale. The model will therefore simulate three to five notes in one upward slur, depending on crooking. The simulation runs over two seconds which is a realistic time for a human player to play such an upward slur.

3. Results

3.1 BIAS

3.1.1 Mouthpiece

The mouthpiece used is a 20th-century mouthpiece by Couesnon, Paris. Its input impedance curve was measured, the peak frequency reveals its mouthpiece resonance frequency (Fig. 5). The peak frequency lies at 613 Hz. There is also a second peak at 3798.5 Hz which corresponds to the resonance frequency of the backbore of the mouthpiece. Fig. 6 shows how the input impedance of a horn compares to the input impedance of a mouthpiece.

The mouthpiece has an effect on the input impedance of the whole instrument as we can see with an example from 204. In its lowest crooking in B basso, the input impedance curve shows 21 clear peaks and another six weaker peaks (Fig. 7). It appears that the cut-off frequency of the instrument – the frequency above which standing waves will be attenuated - occurs after the 19th peak as the 20th peak diminishes in magnitude. But the 21st peak is stronger than the 20th, it is in fact a peak at 617 Hz! By being so close to the input impedance peak of the mouthpiece (613 Hz), this peak is strengthened as the instrument itself cannot sustain a strong peak above its cut-off frequency. This can be seen in all crookings, where some frequencies above the cut-off frequency of the horn are still substantial enough to play well as they lie in the area of the mouthpiece resonance frequency. Fig. 8 illustrates how the peak at 620 Hz is still clearly visible due to its proximity to the mouthpiece resonance frequency; the two next peaks lie considerably weaker and peaks at even higher frequencies are effectively too weak to be considered. The role played by the mouthpiece is crucial as it increases the number of notes the instrument can comfortably produce by multiplying the input impedance of frequencies around the mouthpiece resonance frequency. If the latter lies above the cut-off frequency, otherwise very weak peaks become playable.

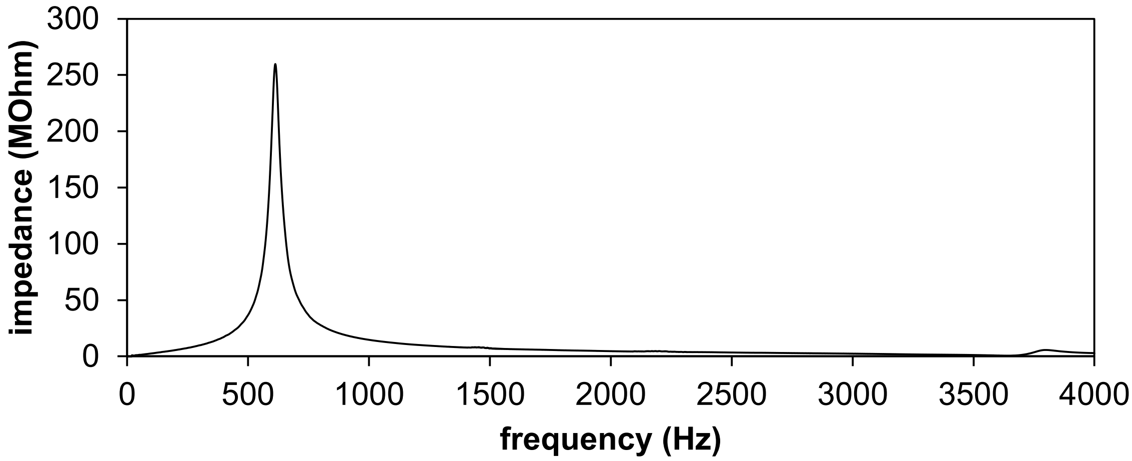


Fig. 5: Input impedance of mouthpiece (AM 1266).

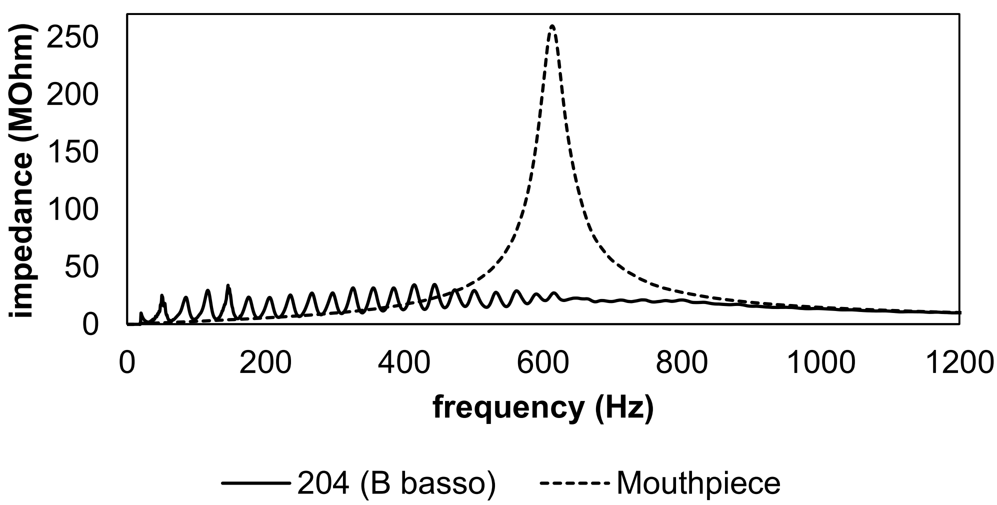


Fig. 6: Input impedances of 204 crooked in B basso and the mouthpiece AM 1266.

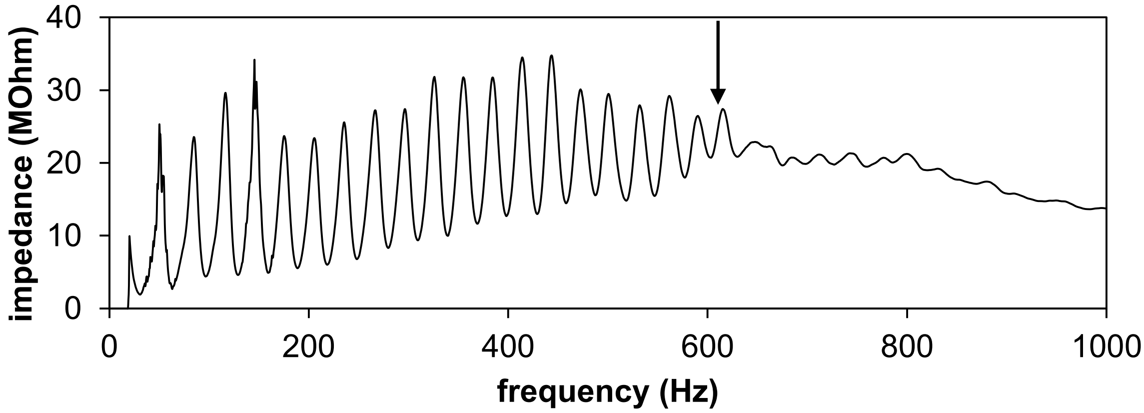


Fig. 7: Input impedance of 204 crooked in B basso, arrow indicates the mouthpiece resonance frequency (613 Hz).

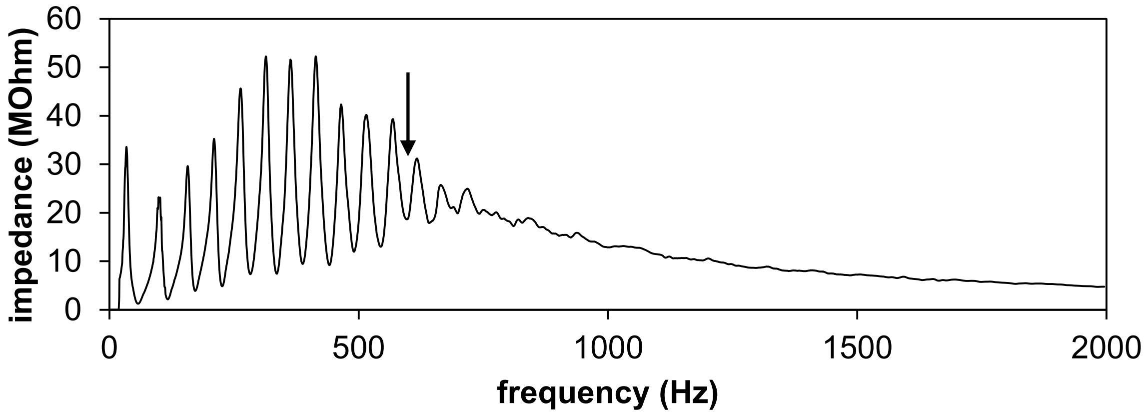


Fig. 8: Fig. 4c: Plot of input impedance of 204 crooked in Ab, red arrow indicates the mouthpiece resonance frequency (613 Hz).

3.1.2 1874

The Mahillon instrument has terminal crooks for E, F, G, A and Bb. There are two crooks for G, three crooks for F and the possibility of crooking E with an F crook and a semi-tone coupler. The original crooks which came with the instrument are darker in colour, the newer crooks by Hawkes & Son are the lighter in colour ♭F, # F and G crooks. Overall, the instrument seems to produce a rich sound as the harmonicity plots line up well. We can see for example that the original F crook (A=458) produces a harmonicity plot on which the 8^{th}, 10 and 12^{th} harmonics will resonate with the 4^{th} harmonic. The 7^{th}, 9^{th} and 13^{th} harmonics resonate with the 5^{th} harmonic. The instrument should still produce a rich sound when lipping notes up or down as there is a good chance that the new note couples with a different oscillatory regime.

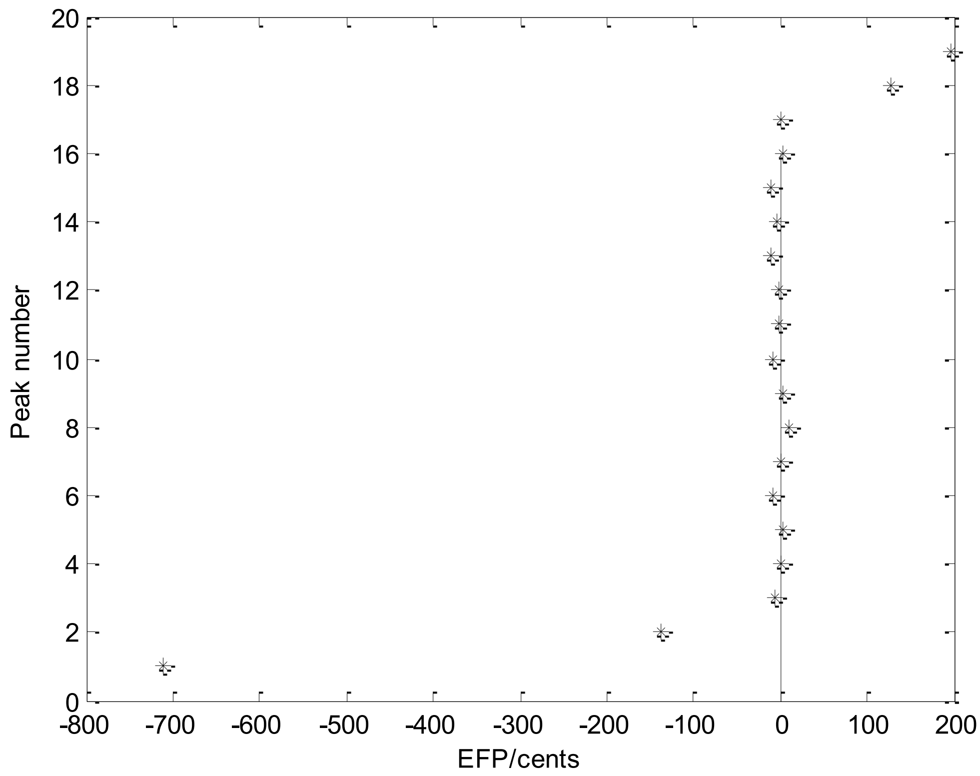


Fig. 9: EFP of 1874 crooked in F (original crook).

Examining the input impedance of the three F crooks gives indications as to why a number of crooks were needed for the same key. The ♭F crook is tuned to A=440 while the #F and the original F crooks are tuned to A=458. We find that both sharper crooks are almost identical in tuning (Tab. 3). They work reasonably well with the A=458 tuning standard with major discrepancies only on the 11th and 13th peak where they are very sharp and flat respectively; this is usual in horns (Norman 2013). The difference between the two sharper F crooks lies in their timbres. The newer crook has a darker sound as lower resonances create oscillating regimes (Fig. 10). The original crook has a more brilliant sound over the whole range as a number of resonances align on the EFP plot (Fig. 9). The b F crook very satisfactory tuning in A=440 widens the range of uses for the instrument, when this tuning standard is needed (Fig. 11). The largest discrepancies appear at the 11th and 13th harmonics which are usually a bit sharp and flat respectively, which is a usual occurrence in horns. The first harmonic is very flat with all crookings: as much as 6 semitones (30 Hz rather than 43-45 Hz)!

Peak Number	F original	#F	A=458
1	30.5	29	45
2	85	88.5	90
3	137.5	138	136
4	184	185.5	181
5	230.5	233	229
6	274.5	278.5	272
7	322	324	323
8	370	371	363
9	414.5	416	408
10	458	458.5	458
11	505.5	506.5	485
12	551.5	551	544
13	594	594.5	611
14	642.5	642.5	647
15	686	683	686
16	737	733	727

Tab. 3: Plot of frequency of harmonics in A=458 tuning against the frequency of harmonics measured on 1874 crooked in # F and F.

The harmonicity plots tell us that the newer ♭F and #F crooks produce a darker sound in the lower frequencies as they do not couple with the higher resonances. The original crook has a different sound as more of its resonances across the range align, for example the 4th, 11th, 12th and 16th; the sound is fuller and brighter. By using the two Hawkes & Son crooks, the player achieves a better tuning but also a less full sound. This is indeed a compromise one might have to make when playing in an orchestra, where the brass instruments should not overpower the orchestra anyway.

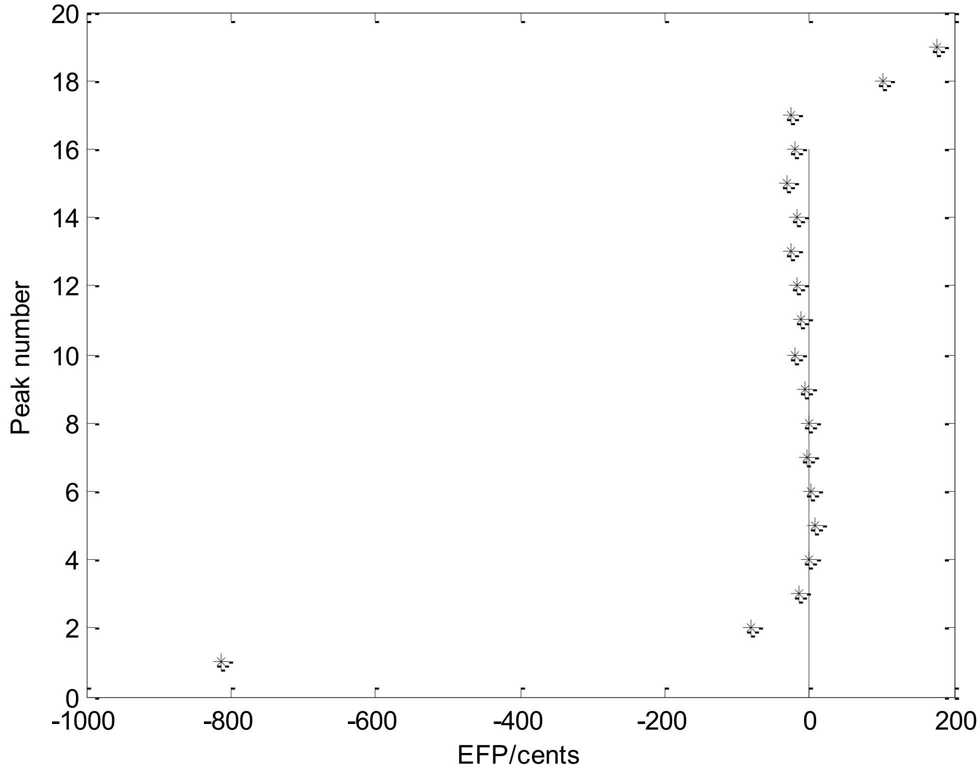


Fig. 10: EFP of 1874 crooked in F (#F).

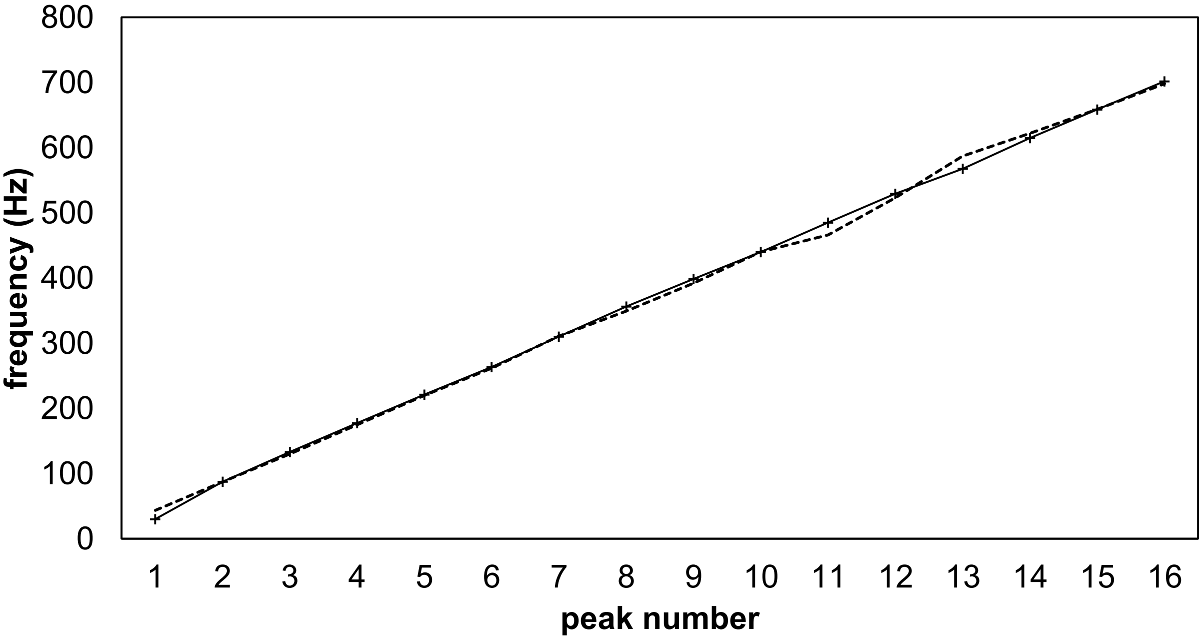


Fig. 11: Frequencies of input impedance peaks measured on 1874 crooked in bF and corresponding frequencies expected at A=440 tuning standard, dashed line = 1874 and straight line = A=440.

An inspection of the two G crooks reveals another interesting consideration (Fig. 7). The original crook tunes the instrument at A=458 whereas the newer one tunes it at A=452. Effectively this means that the 8th harmonic (here of course a G) sounds at 458 Hz and 452 Hz respectively. But up to this note, both crooks produce almost identical peak frequencies and the 'lower' crook is sharper than the original crook up to the 7th harmonic. It is after that the tuning of the two crooks begins differing more sharply; there is a difference of 53 cents on the 16th harmonics. As a consequence, the two G crooks are almost equal in tuning at low range.

Keeping in mind that the difference between the crooks is not necessarily huge in terms of tuning, their tone quality can again be a big factor in choosing one. We observe that the original G crook has a good harmonicity between the 6th and 14th harmonics, resulting in a brilliant sound. The newer crook shows a marked difference between its lower and higher range, with harmonics aligning roughly below and above the 8th harmonic (Fig. 13). As with the the newer sharp F crook, this crook produces a dark sound in its lower range and a more brilliant in it upper range.

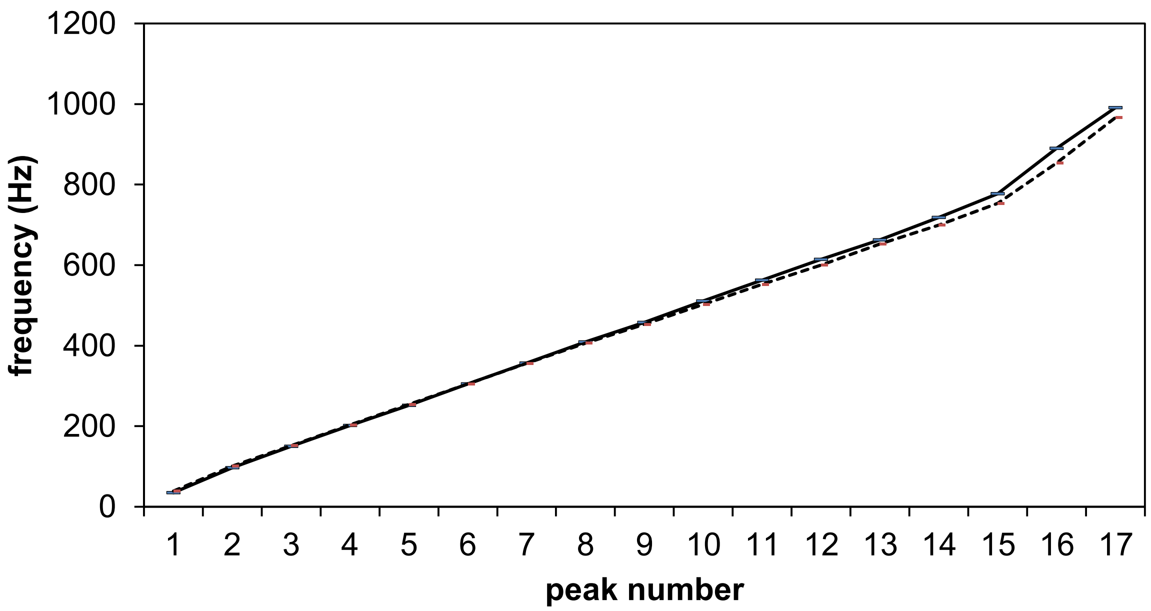


Fig. 12: Frequency of harmonics for 1874 crooked in G, straight line = original crook at A=458 and dashed line = newer Hawkes & Son crook at A=452.

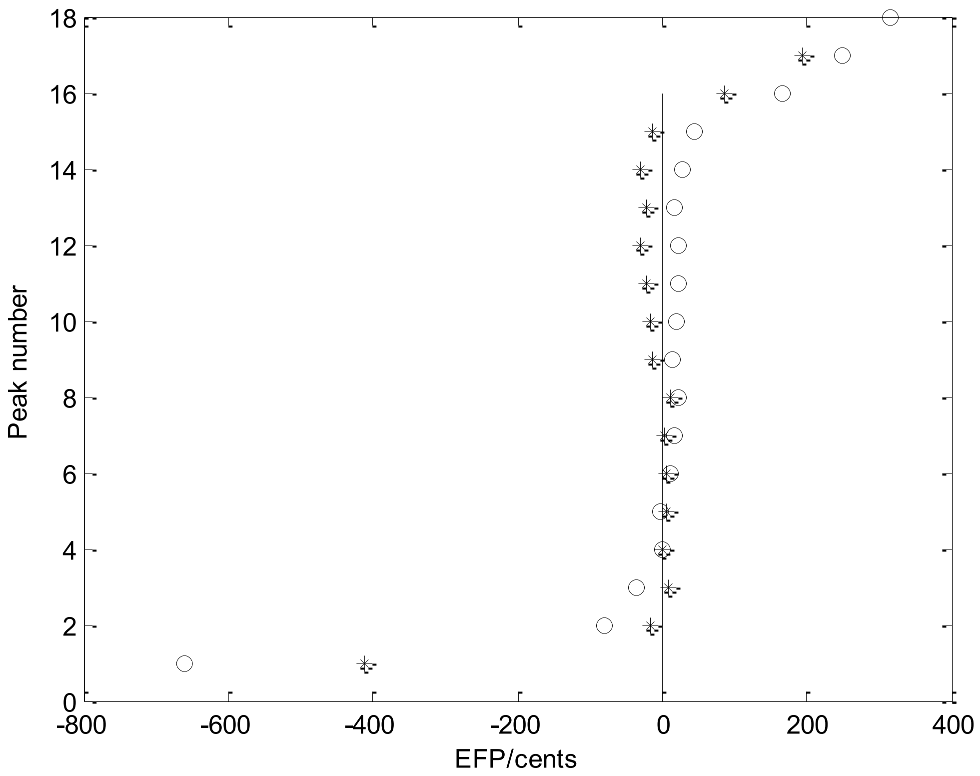


Fig. 13: Plot of EFP of 1874 crooked in G, circles = original crook and crosses = newer Hawkes & Son crook.

3.1.3 Inventionshorns

We measured two inventionshorn type instruments: 533 and 6144. In the catalogue, 6144 is labelled as a 'cor solo' but its crooking system is identical to 533. Cor solo is the name the maker Raoux gave these instruments as they are particularly adapted to playing on their own. The crooks which come with the two instruments are the ones for the most common key; 533 has Eb, E, F, sharp F, G and 6144 has D, Eb, E, F, G. The limited range in keys confirms that the cor solo is not meant to play in orchestras which might play pieces in keys further removed. This is less of an issue for 533 which has three valves.

The 533 horn by Boosey and Co. displays a tendency to go flat above the 4th harmonic and sharp above the 14th harmonic (Fig. 14), as this behaviour can be observed for every crooking. We have seen that this is because of the rapidly flaring shape of the bell. As a consequence, little vibration coupling as few resonance modes align. The 2nd, 3rd and 4th harmonics tend to vibrate sympathetically, unfortunately a look at the input impedance shows that lower harmonics have rather low and wide peaks, rendering them difficult to play precisely (Fig.

15). The frequencies that have higher and narrower peaks are also rather flat and struggle to align with any other standing waves. The player will need to tune the notes, particularly when playing in an ensemble.

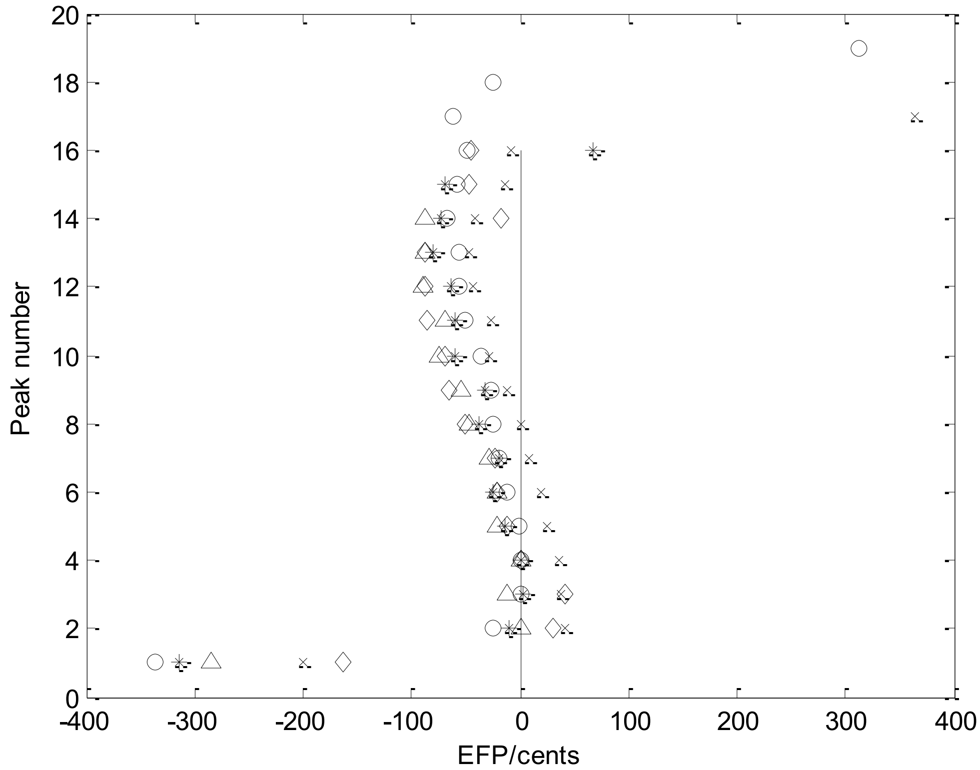


Fig. 14: EFP of 533 crooked in Eb, E, F, sharp F and G. Circle = Eb, star = E, cross = F, triangle = sharp F, diamond = G.

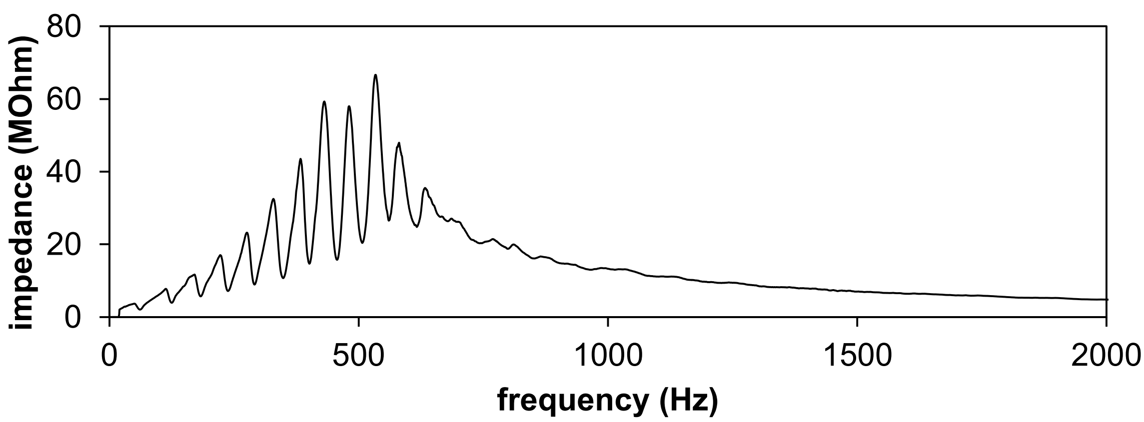


Fig. 15: Input impedance plot of 533 crooked in G. The lower peaks are particularly large and small in amplitude.

The cor solo by Raoux shows a similar response in all five crookings (D, Eb, E, F, G). We see that the second, third and fourth harmonics are in tune, before the resonances flatten as they rise (Fig. 16). The harmonicity plots of both instruments show the same effect, suggesting that the cylindrical section in the middle of the tubing might have a similar effect on the intonation of the horns as their bells. All five crooks for 533 have been found to be almost cylindrical, allowing for the tubing's tendency to become a bit oval-shaped over time. We also find that the crooks are narrower than the preceding tubing 0.6 - 1 cm depending on the crook. The difference with the following tubing is of 0.5 – 0.9 cm. This bore shape hardly approximated the ideal cone, resulting in harmonicity plots with the trends we have discussed.

The catalogue entry for 533 describes the instrument as 'stuffy' (Myers 2006, 32), this could be due to an obstruction in the tubing or a more general construction problem. It is difficult to conclude on the reason the instrument has this playing characteristic from the measurement of its input impedance only.

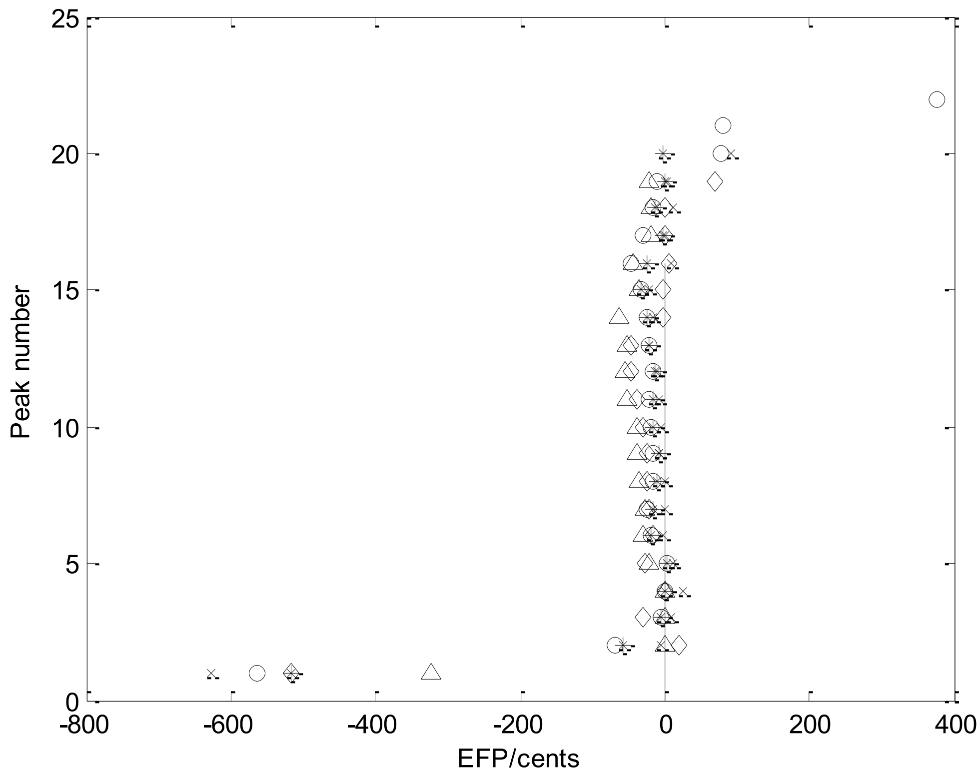
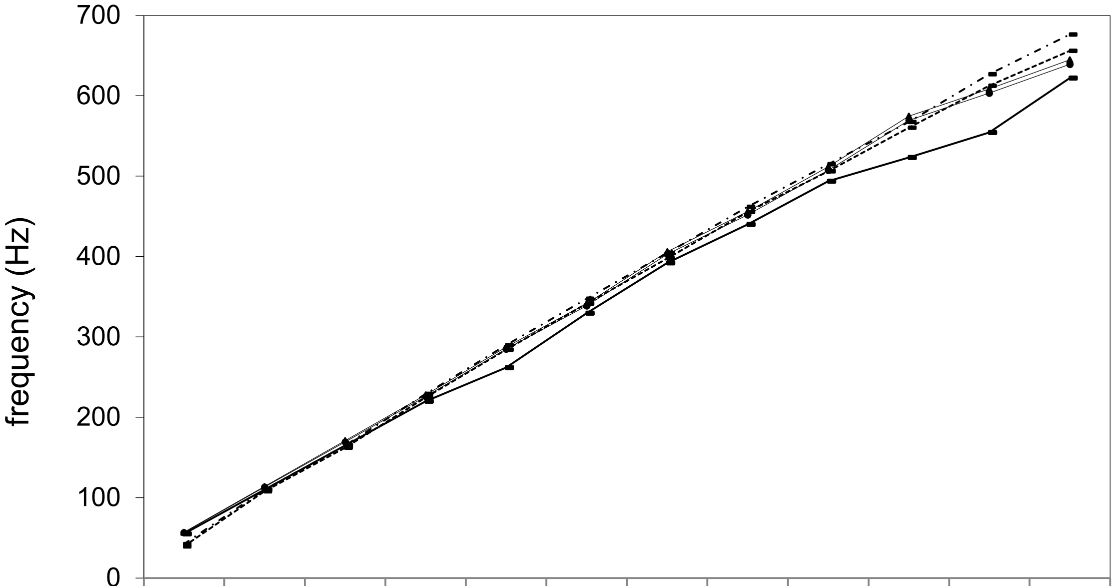


Fig. 16: EFP of 6144 crooked in D, Eb, E, F, and G. Circle = D, star = Eb, cross = E, triangle = F, diamond = G.

3.1.4 203

The Sandbach orchestral horn has a number of master crooks and couplers which can be inserted into the corpus to achieve the length necessary to play in the keys of C basso, Db, D, Eb, E, F, G and A and Bb. The keys of F and A have two possibilities of crooking each, where different combinations of tubings can be used to achieve the length required. The crook combinations measured with BIAS are C, D, Eb, E, F (one possible crooking), G, Ab and A (two possible crookings). As mentioned previously, measuring horns with master crook and couplers proves problematic as the shape of the master crook does not fit well with the shape of the measuring head. Thankfully, it was possible to fit the mouthpiece into the instrument well enough to produce viable BIAS measurement results. A second issue is that some crooks could not be found so that some crooking combinations were not possible.

The EUCHMI catalogue states that the instrument's tuning approximates A=440. The input impedance curves show that this indication could be misleading. If we take the example of the two possible A crookings (*203 c and h, or 203 c and i*), we observe that the first four harmonics for each do indeed approximate A=440. Higher harmonics go distinctively sharp though, suggesting that the instrument is rather tuned A=452 or A=456. In fact, it seems that the two crooking combinations are closer to A=456 as a number of notes correspond to the expected frequencies for this tuning (Fig. 17). This example illustrates why the player needs to compromise when choosing the right crooking, at which they will be able to adjust the tuning of individual notes best. In fact, both combinations produce notes which are in tune and some which are not. Arguably, the 4th harmonic for A (*ch*) sounds at 229 Hz which is close to the ideal 228 Hz; but the the 8th harmonic for A (*ci*) sounds at 455.5 Hz – almost a perfect 456 Hz- whereas A (*ch*) sounds sharp at 462 Hz.



	1	2	3	4	5	6	7	8	9	10	11	12
A (ch)	42	110	165	229	289.5	347.5	404.5	462	515	567.5	627	676.5
A (ci)	39.5	108	162	224.5	284	341.5	397.5	455.5	506	560.5	612.5	656
A=440	55	110	164.81	220	261.63	329.63	392	440	493.88	523.25	554.37	622.25
A=452	56.5	113	169.3	226	284.7	338.6	402.7	452	507.4	569.5	603.3	639.2
A=456	57	114	170.8	228	287.3	341.6	406.2	456	511.8	574.5	608.7	644.9

Fig. 17: Frequencies of input impedance peaks measured on 203 crooked in A (ch and ci crookings), and the frequencies expected at the tuning standards A=440, A=452 and A=456.

A look at the plots of the EFP for all the crookings on the Sandbach instrument reveals two distinctive shapes appearing. The higher keys of A and Ab have EFP plots where the 4th and 9th harmonics align, the harmonics in between these go sharp, the ones above the 9th go flat (Fig. 18). The lower keys of C, D, Eb and E tend to go a bit sharp above the 5th harmonic before re-aligning with the lower harmonics above the 12th (Fig. 19). We can hypothesise that these two trends are due to the fact that the higher keys use different couplers and master crook to the lower keys: *203 c, j, h, i* and *203 a, d, e, f, g*.

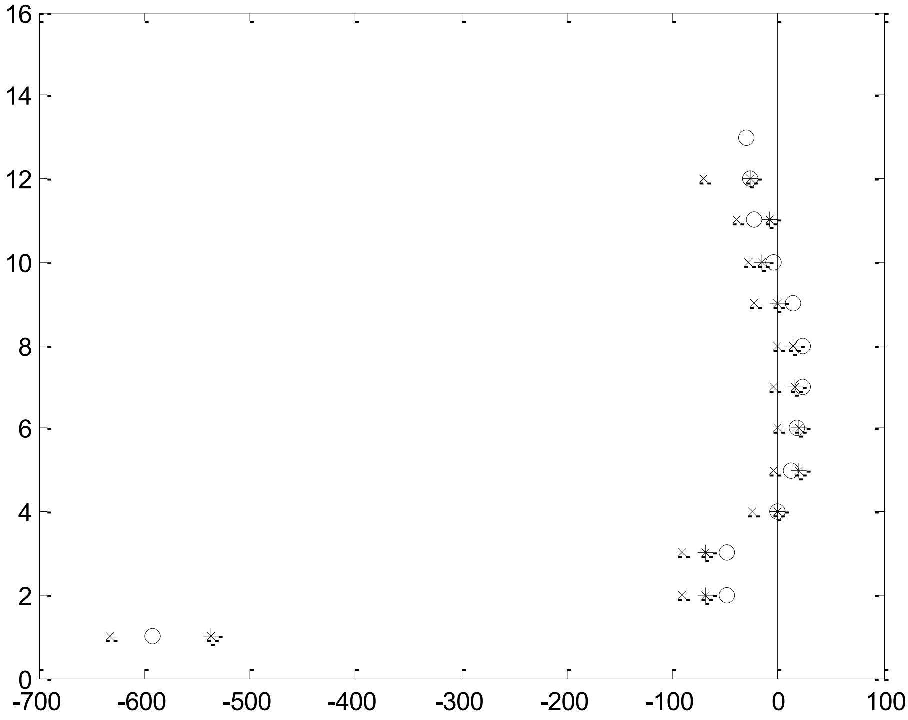


Fig. 18: EFP of 203 crooked in Ab and two combinations for A. Circle = Ab, star = A (ch), cross = A (ci).

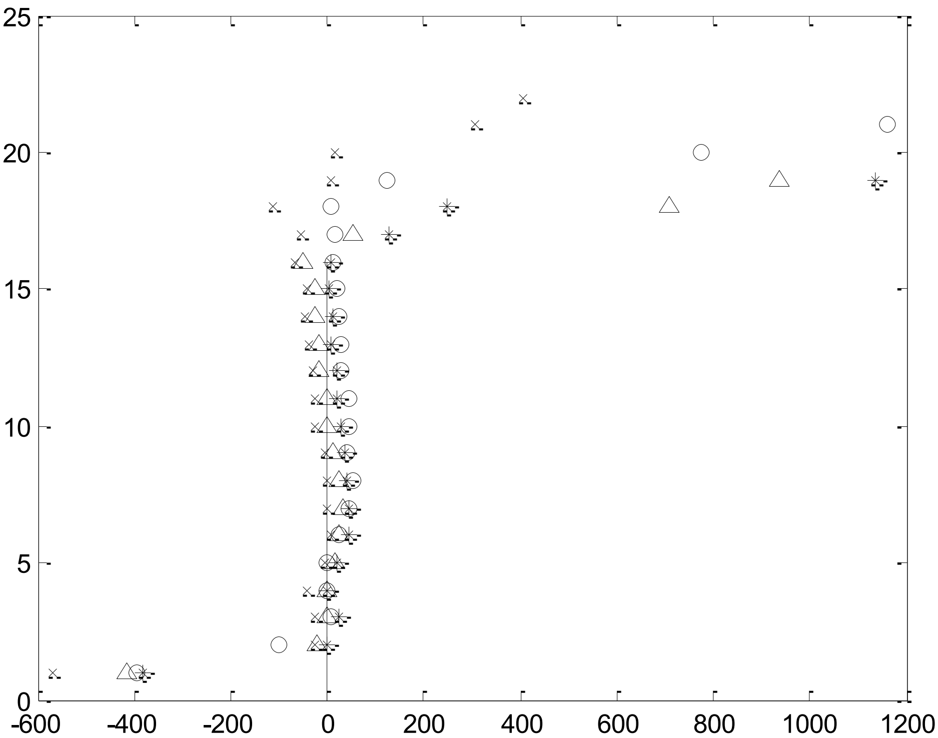


Fig. 19: EFP of 203 crooked in C, D, Eb and E. Circle = C, star = D, cross = Eb, plus = E.

We must also discuss the 'mediocre' results for the instrument crooked in F and G. It is difficult to judge whether the F crooking produces a good sound as its input impedance curve is very irregular. The first ten peaks are visible but very round, the peaks above cannot be read (Fig. 20). We would need to know the reason why this has happened to determine the sound quality of the crooks. Considering that the combination of *203 a and e* are also used for the key of Eb – which works well – it is unlikely that the problem is a hole in the tubing. Using the first ten peaks to calculate its EFP produces an acceptable result; we are therefore tempted to put this odd result down to an error while measuring. The G crooking produces an input impedance curve similar to the rest of the measurements of this instrument. However the EFP plot shows large disparities in harmonicity (Fig. 21). Again, it is difficult to say whether this is due to the physical properties of the crooking or an error in measurement.

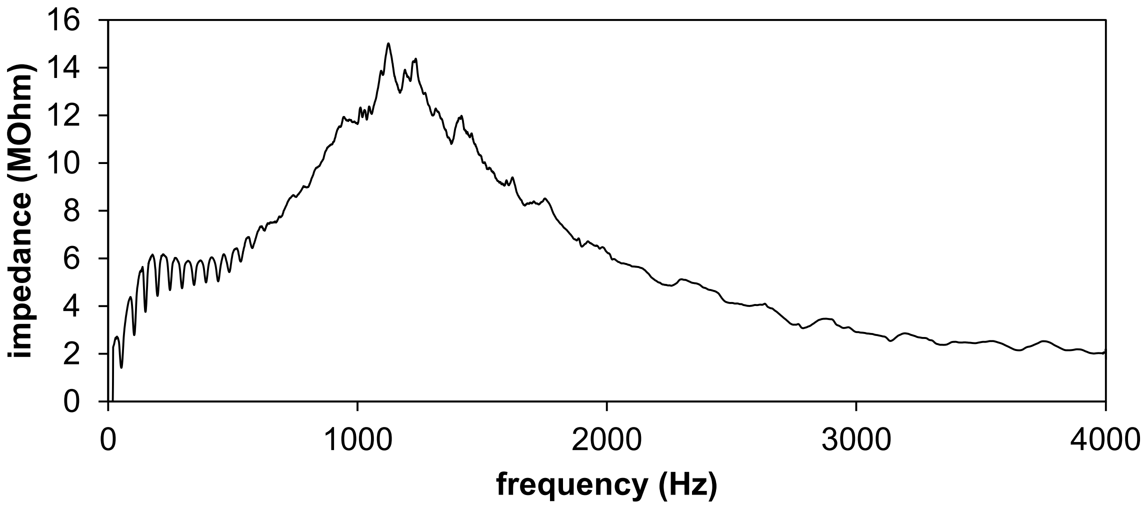


Fig. 20: Input impedance curve of 203 crooked in F.

A possible reason for some of the results we have seen is the origin of the bits of tubing. The catalogue tells us that some pieces are of different workmanship: *203 a, b, g and j*. Little more detail is given which means any assumptions about the effect of different workmanship on the input impedance of the instrument would be speculative. Further investigations could look at whether *203 a* is in fact not the appropriate choice of master crook for the Sandbach instrument, this idea is supported by the fact that the crooking in G, which only uses this crook, has a messy harmonicity.

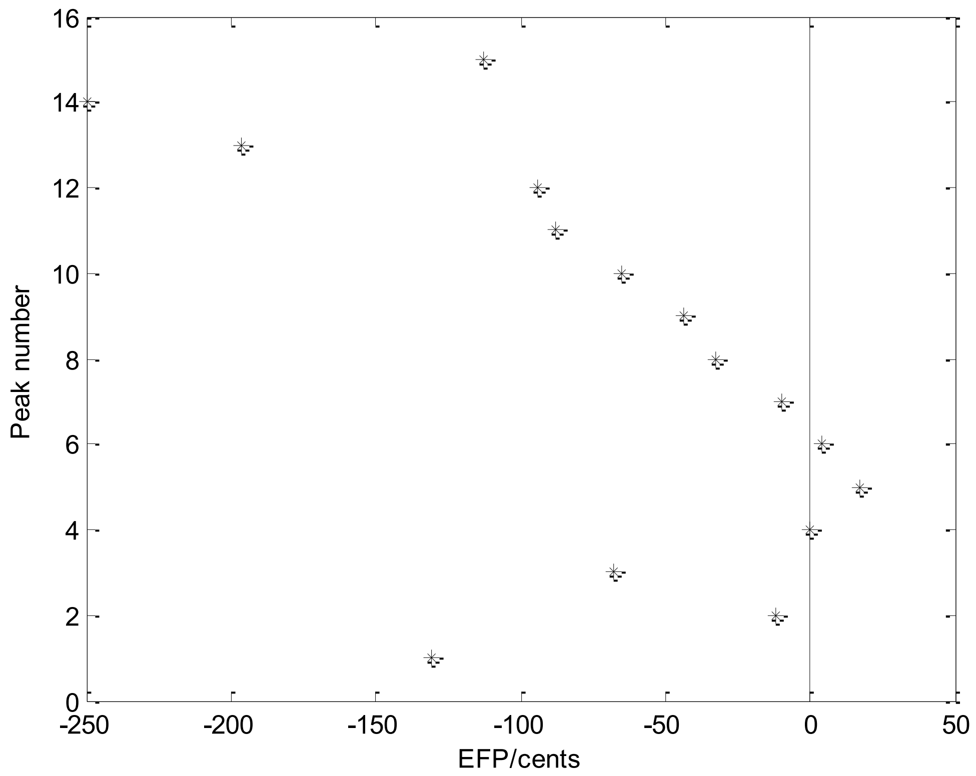


Fig. 21: EFP plot for 203 crooked in G.

It becomes apparent how much influence the crooking chosen can have on the sound and the playability of the instrument. It is therefore not unusual to see different combinations for the same key as they can make a big difference to how the instrument behaves. This knowledge needs to be balanced with the ergonomic problems that couplers can present. We have found that the lower keys are most convincing in terms of harmonicity, but these also demand the most number of couplers at once (a master crook and four couplers for C basso!). Such a construction is obviously more impractical as the instrument is so far removed from the player's body; precise lip work becomes more difficult too.

3.2 Modelling

3.2.1 BIAS

The BIAS software produces an input impedance curve calculated from the physical dimensions of the horn provided. It can then be treated in the same manner as the measured input impedance curves.

The modelled EFP plot for the inventionshorn 533 shows excellent harmonicity for the E and sharp F crooks. The Eb, F and G crooks are still surprisingly aligned. It is striking that these EFP plots lack the distinctive shape of the measured results, where the resonances flatten as they rise. The first resonance is far flatter for the modelled results than for the measured results (Fig. 22). It appears that the discrepancies between measured and modelled results occur where the bell shape affects the input impedance of the instrument. Thus the first resonance and the flattening effect in rising resonances are not well represented. It is possible that the BIAS model struggles to deal with the effect of the bell shape on the acoustics of the horn. It is also debatable whether the cylindrical sections in the inventionshorn, the crooks, are well simulated in BIAS. They contribute to a misalignment in harmonicity, as is usual for hybrid bore shapes; but it is unclear how much it affects the modelled input impedance.

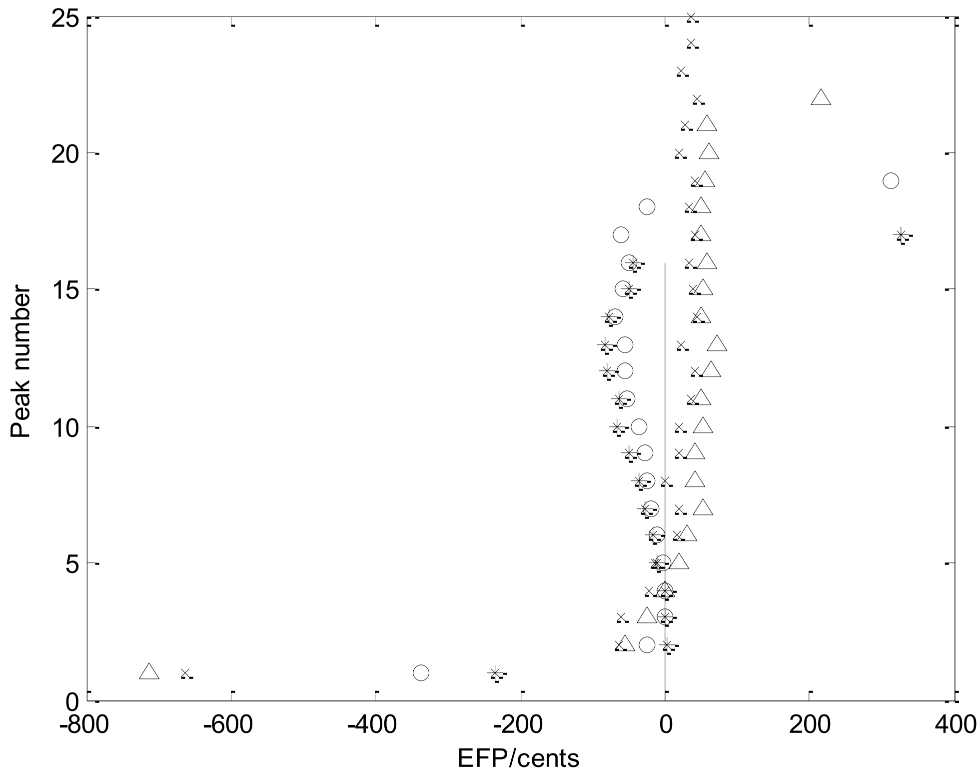


Fig. 22: EFP of 533 crooked in Eb and G, modelled and measured. Stars = 533 in F (measured), circles = 533 in Eb (measured), crosses = 533 in F (modelled), triangles = 533 in Eb (modelled).

Another distinctive feature of all results produced by BIAS is the amount of input impedance peaks. It is usual to see 25-30 peaks for each crooking whereas measured instruments rarely surpass 20 clear peaks. A look at a modelled input impedance curve shows a cut-off frequency above which the peaks weaken, but clear enough peaks are produced at high frequencies to be processed by our Matlab peak detection programme as such (Fig. 23). It would appear that the model simulates an instrument in which little losses are taken into account. In fact, it would be impossible for a real brass instrument to radiate enough energy at 5000 Hz for example to produce such an input impedance curve.

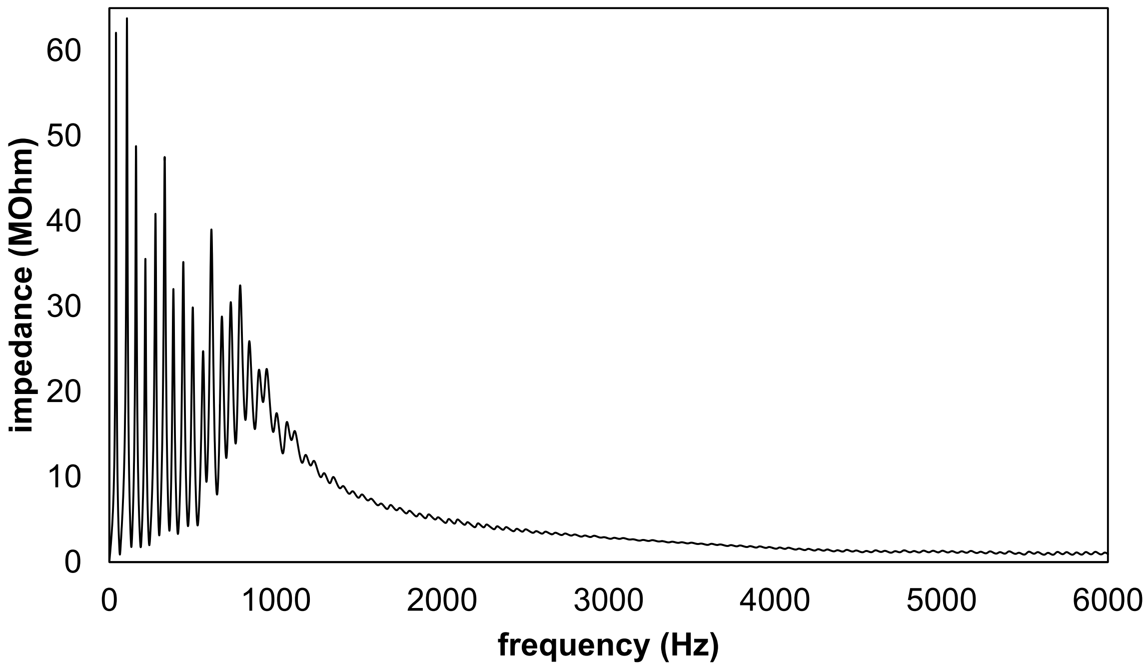


Fig. 23: Input impedance curve of 533 crooked in G modelled by BIAS.

Damping can be simulated in BIAS. This can be caused by friction at the walls, thin walls and leaky valves. (Widholm 2008, 150). The instrument 4092 has a large dent (118 cm long) in the tubing which effectively flattens the side of the bore. The effect of the dent was simulated by using roughness factor of 2 and then 4 when entering the dimensions of the instrument; keeping the rest of the bore at the suggested roughness factor of 1. Note that the diameter of the bore does not change at the bore, we rather change the roughness factor of the patch. This affects the damping rather than the intonation of the instrument. The simulation shows that the peaks up to around 450 Hz for both instruments with increased damping are stronger. Above 450 Hz they then become weaker than for the instrument with no damping. It is equally visible that the low peaks for the instrument with higher damping are stronger, then weaker than for the instrument with less damping (Fig. 24 and 25). The curves of the damped instrument look closer to the measured curves. Particularly the cut-off frequency appears more realistic as the amplitude of the peaks above it drops off more sharply. We also recognise the shape of peaks in the area of the mouthpiece resonance frequency as they would look on measured curves. This result is intuitive as real instruments will most often have some sort of damping due to tear and wear or building imperfections. Nevertheless, the amount of energy radiated above 1000 Hz is still higher than in real instruments.

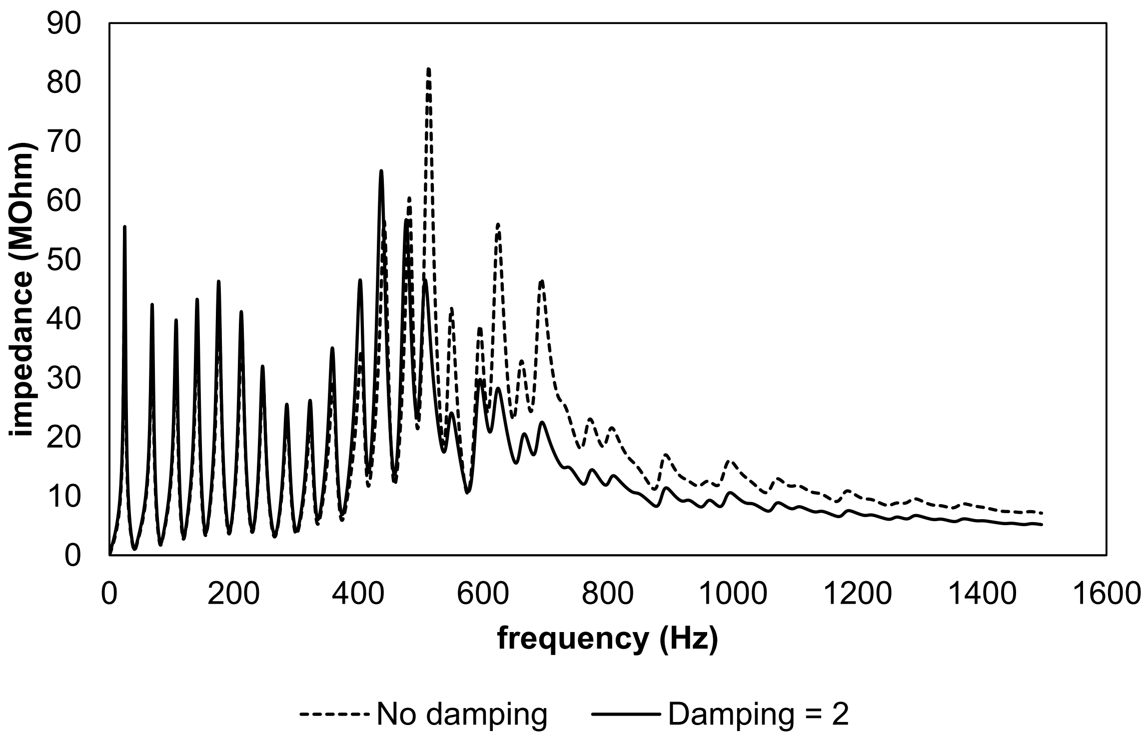


Fig. 24: Modelled input impedance curve of 4092 crooked in Eb with a damping factor of 2 for the length of the dent in the tubing.

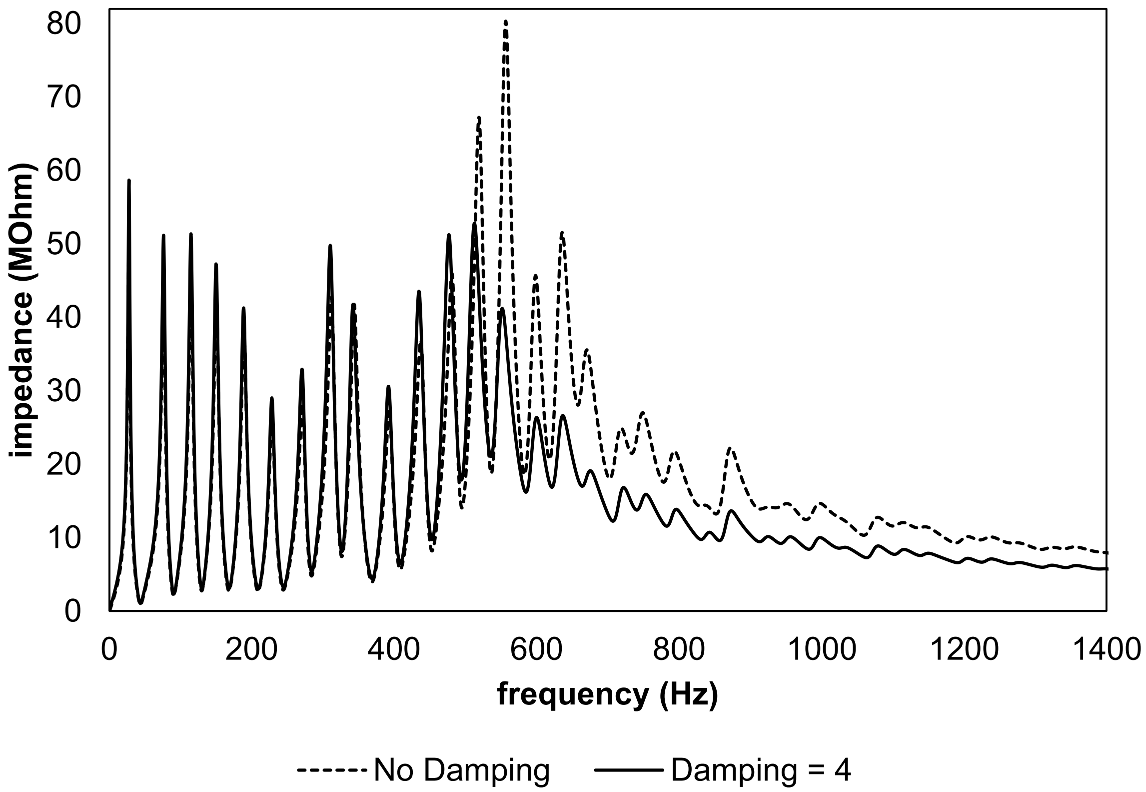


Fig. 25: Modelled input impedance curve of 4092 crooked in F with a damping factor of 4 for the length of the dent in the tubing.

The Eb crook for the 4092 instrument has a **hole** in the pipe, the measured input impedance is therefore misleading as energy is dissipated and the standing waves that build up in the instrument are not harmonic. An attempt to cover up the hole with some taper and fabric produced a measurement, but comparison with measurements of the other crooks and the BIAS simulation reveal that this method does not solve the issue. We can deduct that the Eb would therefore be unplayable unless it was repaired properly or replaced. Fig. 26 shows that the measured peaks are further apart than the modelled peaks, and the measured curve is broken up in places which points at issues with the instrument.

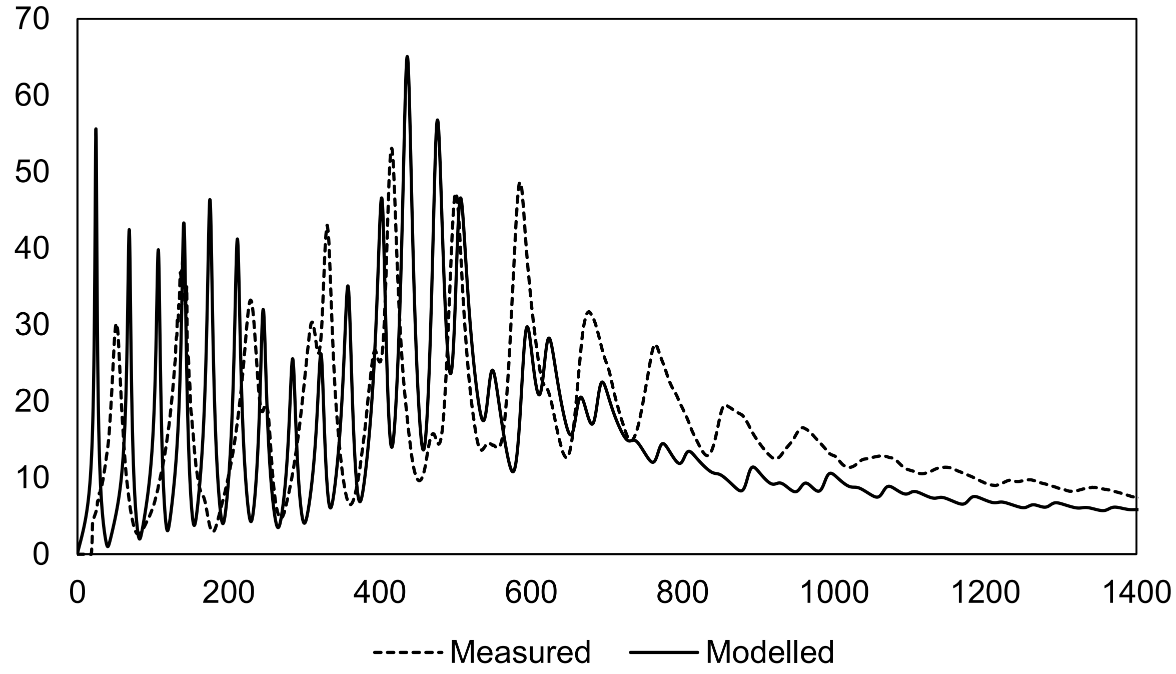


Fig. 26: Input impedance of 4092 crooked in Eb, measured and modelled. The Eb crook has a hole in the pipe.

As discussed earlier, the measurement of the instrument **203** with the BIAS equipment was difficult as the shape of the master crook prevents it from completely entering in the measuring head. The modelling of the input impedance can help discuss the intonation of the instrument, as we established that the measured results strayed far from the suggested A=440. The simulation results are consistently lower than the measured results. Particularly the D crooking approximates the A=440 tuning well, while other crookings are close enough (Tab. 4). We could argue that the simulation gives a more accurate indication of the instrument's tuning than the input impedance measurement. This is not necessarily because the effective length of the tubing is extended when using BIAS because the instrument does not completely enter the measuring head. The problem is rather that even when the instrument is held straight to measure with BIAS, there is a possibility that it wobbled; the result is then distorted. Even though the modelling results appear to be more accurate for 203, one needs to be careful still as we discussed the model's difficulties with simulating the effect of the bell on input impedance. Fig. 27 shows the modelled EFP for the D crooking; a series of resonances that close to the harmonic series has not been observed in any instrument. It confirms the idea that the programme does not take into account all factors affecting an instrument's intonation. An obvious issue is the fact that the diameter of the bore gives no indication whether the inside of the tube presents any irregularities. These are difficult to spot when using our method for measuring tube diameter, particularly because the bent shape of the horn makes an internal inspection of the tube impractical.

D measured	D modelled	A=440	Eb measured	Eb modelled	A=440
30.5	23.50392	36	30	25.50425	38
76	69.51158	73	82	75.51258	77
115.5	110.0183	110	123	118.0197	116
152	146.5244	146	162.5	156.5261	155
192	185.5309	185	207.5	199.5332	196
234	223.0372	220	251	238.5397	233
273	259.0432	261	291.5	276.046	277
311	295.0492	293	333	318.053	311
349.5	334.0557	329	373.5	357.0595	349
386.5	371.0618	369	410.5	396.5661	392
422.5	407.5679	392	451	436.0727	466
461.5	445.0742	440	490.5	479.5799	493
496.5	485.5809	493	529	514.5857	523
535.5	518.5864	523	567	554.0923	554
571.5	555.0925	554	609	599.5999	587
610.5	596.0993	587	641	647.6079	622
F measured	F modelled	A=440	G measured	G modelled	A=440
35	30.50508	43	49.5	36.006	49
89	88.01467	87	106	99.51658	98
138	133.5222	130	154	149.5249	146
178	180.5301	174	213.5	203.0338	196
220	226.5377	220	269.5	252.042	246
268	269.5449	261	321	301.0502	293
320.5	315.5526	311	371.5	352.5587	349
371	360.5601	349	419	403.0672	392
414	405.5676	392	468.5	454.5757	440

463.5	452.5754	440	514	503.5839	466
515.5	497.5829	493	558	552.092	523

Tab. 4: Peak frequencies of input impedance of 203 crooked in D, Eb, F and G; the three columns are the measured frequencies, the modelled frequencies and the expected frequencies for A=440.

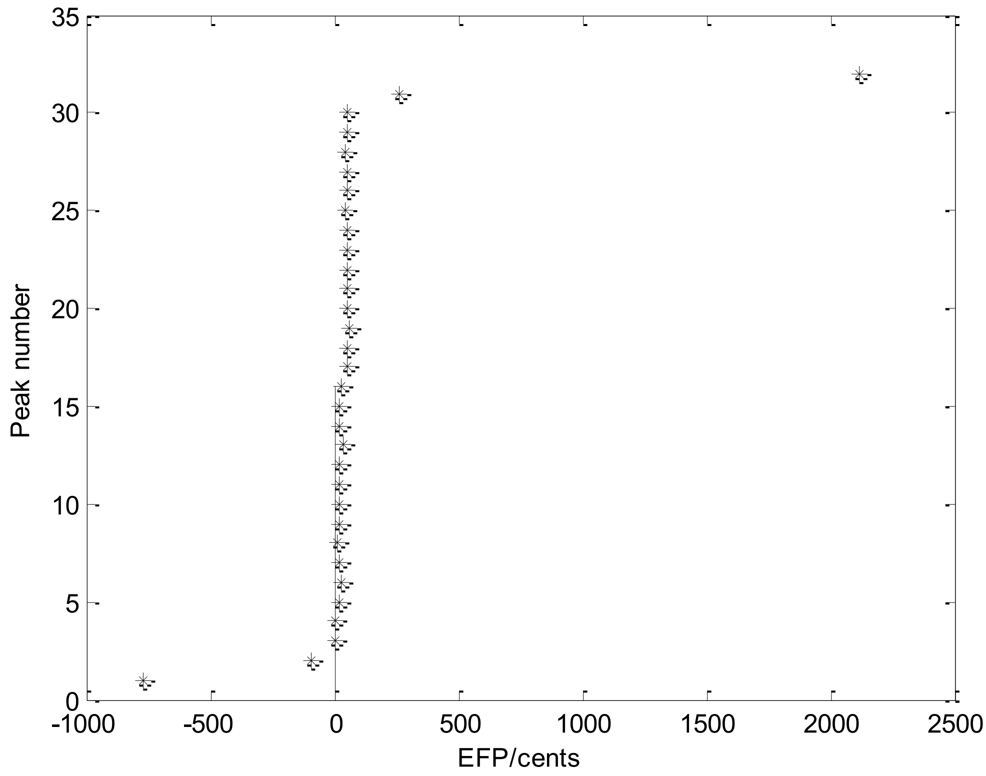


Fig. 27: EFP of 203 crooked in D as modelled in BIAS.

3.2.2 FDTD Simulation

The FDTD model simulates the behaviour of a brass instrument when played at a lip frequency over a certain period of time; in this case the frequency will rise from 300 Hz to 450 Hz in two seconds. The resulting plot shows a sharp rise in the initial attack in the absence of an input impedance peak. The playing frequency then pulls upwards until the next transient where the frequency sharply rises to the next peak. We find that the peak frequency is passed during the upwards bend, lying roughly in the middle of it. The width in cents of the upward bend determines the instrument's capacity for pitch bending (Norman 2013, 537). We can see on Fig. 28 that although the pitch after the transient is slightly below the peak frequency, the pitch is bent far further upwards. The effect is reversed when lipping downwards (Netwon et al. 2014), as the note is bent further downwards. Note that the acoustical resonance frequencies on Fig. 28 are sourced from the results produced by BIAS measurements.

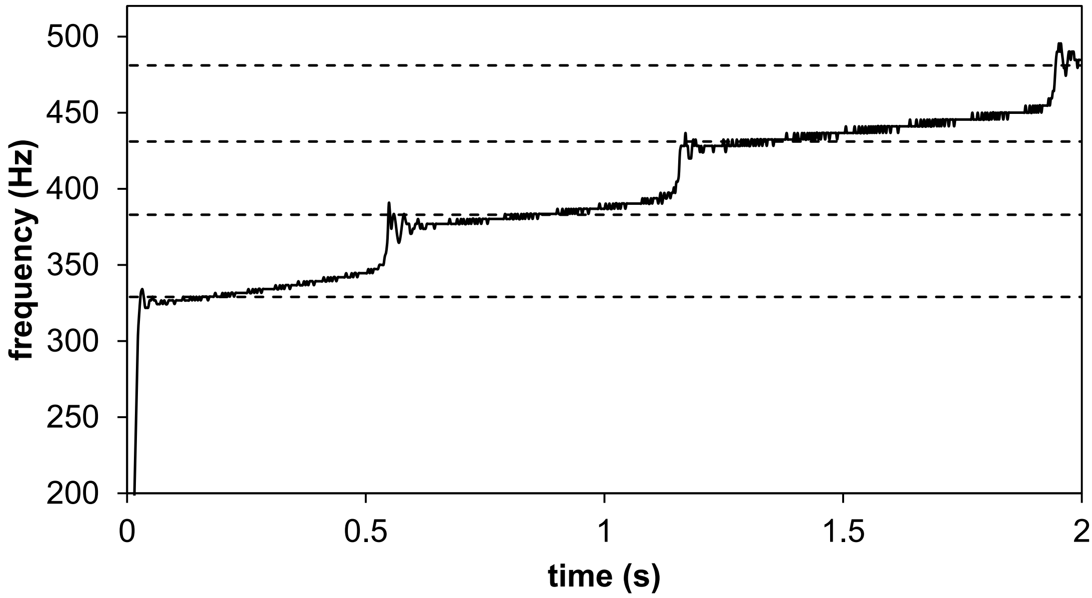


Fig. 28: Simulation by FDTD model of upward lip slur played on 533 crooked in G and lip frequency running from 300 to 450 Hz. Dotted lines are the acoustical resonances as measured on 533 crooked in G by BIAS, these are 329 Hz, 383 Hz, 431 Hz and 481 Hz.

It has been found that for all the modelled results the playing frequency is higher than the lip frequency due to the outward-striking lip red behaviour of the lips (Newton et al. 2014). This effect appears to be far more accentuated in this work as the lip frequency only approximates the playing frequency during the initial transient (Fig. 29). We find a difference of up to 50 Hz between the two frequencies during the transients where they most differ as the note is pulled up the next acoustical resonance.

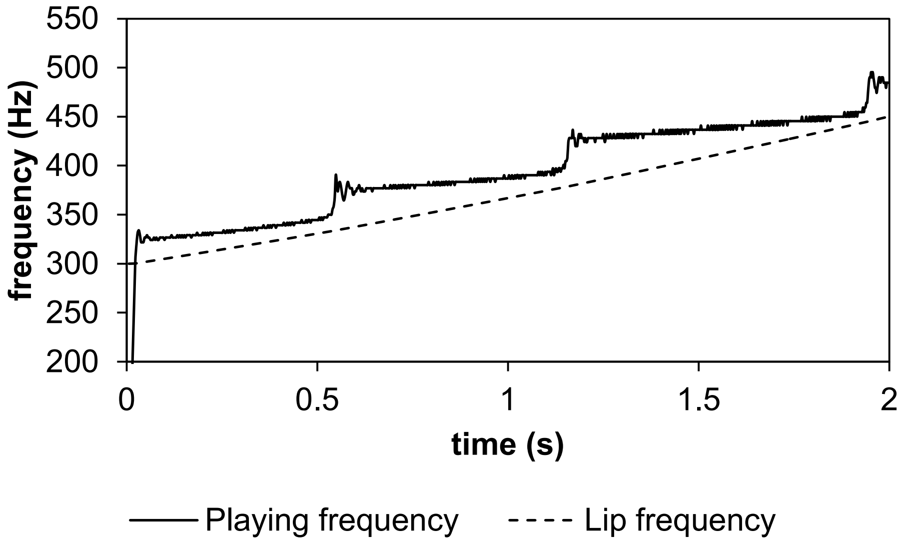


Fig. 29: Simulation by FDTD model of upward lip slur played on 533 crooked in G. The dotted line is the lip frequency used in the simulation.

A comparison of the pitch bending potential for all the modelled instruments shows values between 20 and 150 cents. We find that the shorter the crooking (therefore the higher the key), the larger the pitch bending observed. Fig. 30 shows how far a pitch can be bent in each crooking of 4092. The difference between the starting and end frequency of the note corresponding to an acoustical resonance is measured in cents. We can see that for most resonances the pitch bending potential hovers around 50 to 90 cents, but the F, G and Ab crookings go above 100 cents for some resonances. These results suggest that the crooks not only influence the input impedance of the instrument but also its pitch bending potential. An increased capacity for lipping on shorter crooks would be beneficial considering the experimental results regarding the often worse intonation of crooks such as Ab, A, Bb and B (Fig. 31). It means that although the tuning can be worse on shorter on crooks, there is also more scope for the player to adjust the intonation by lipping appropriately.

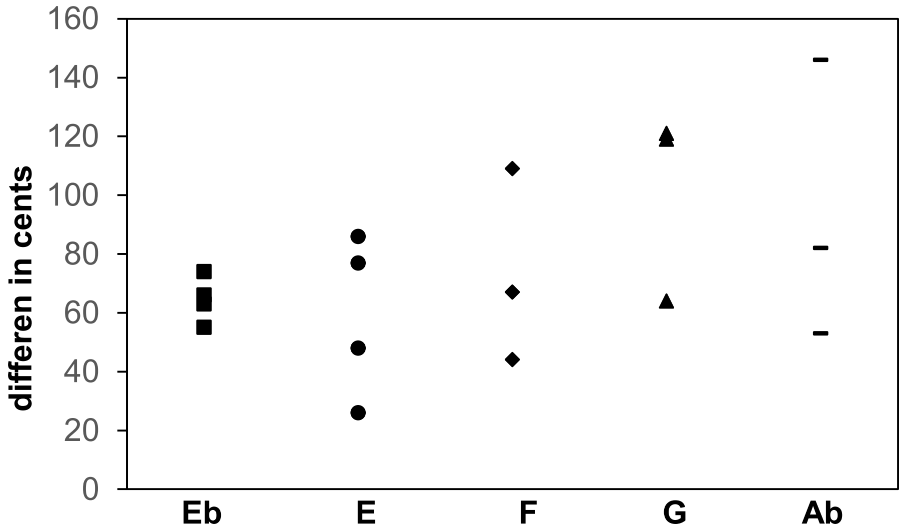


Fig. 30: Pitch bending potential of 4092 in each of its crookings. For each crooking, the data points represent the difference in cents for the each note corresponding to an acoustical resonance during an upward slur from 300 Hz to 450 Hz.

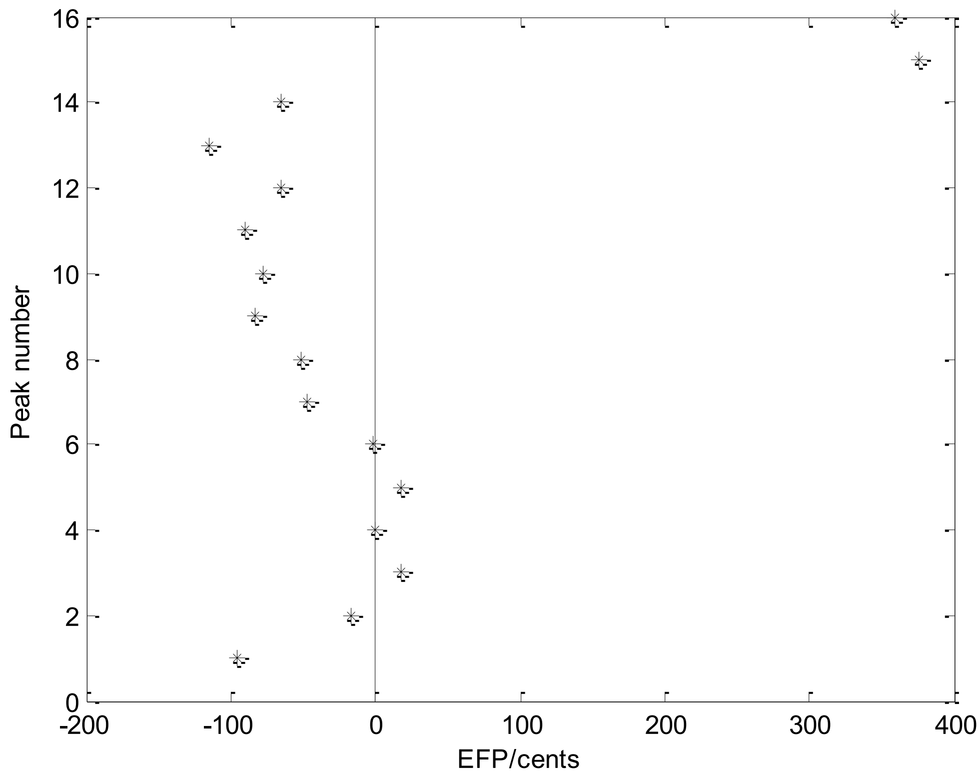


Fig. 31: EFP off 4668 crooked in B measured by BIAS.

4. Conclusions

4.1 Further work

We have found that even though our focus lies on the effect of crooks on horns, it was almost impossible to not take the effect of the bell into account. It would be of interest to look deeper into the coupling of the bell and different crooks. Another area for discussion is the relationship between the bell shape described by the Bessel equation (see 1.3.2) and the sound characteristics attributed to instruments such as ‘German’, Viennese’, French’, ‘dark’, ‘bright’ and so on. The horns measured have in fact been found to have rather distinct bore and particularly bell shapes (Fig. 31-33). Another factor omitted in the discussion is the valve section. As many of the measured instrument were valved, it would be interesting to discover how the valves affect the sound of the instruments.

Further work could also be done on a more extensive amount of instruments modelled. A greater number of results would make for interesting further insight into the coupling of a lip model to horns in different crookings. It could also help discuss instruments that we have not been able to measure with BIAS such 2888 or 3296. Considering the good results of both the BIAS and the FDTD model, we could confidently discuss results for these instruments even though the measurement of their input impedance is not possible with BIAS.

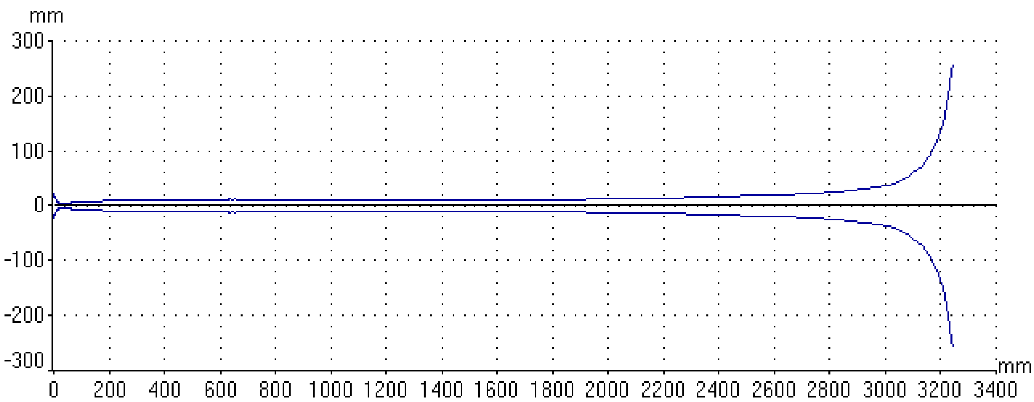


Fig. 31: Bore shape of 203 crooked in G.

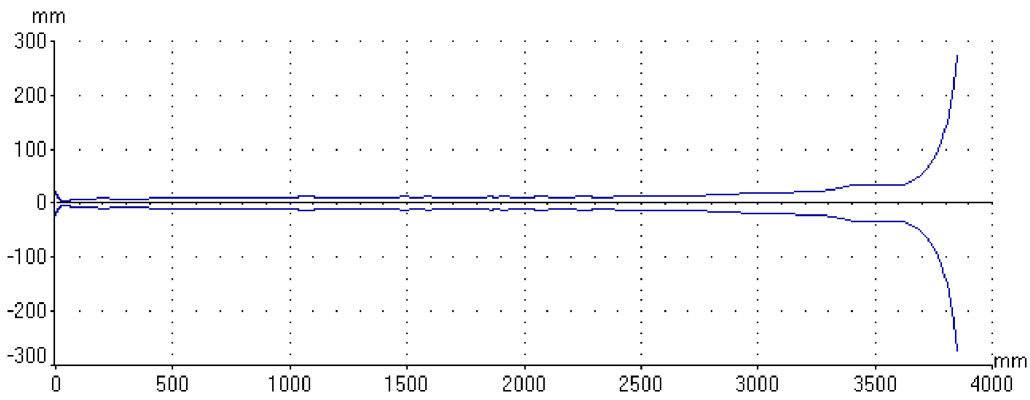


Fig. 32: Bore shape of 4092 crooked in G.

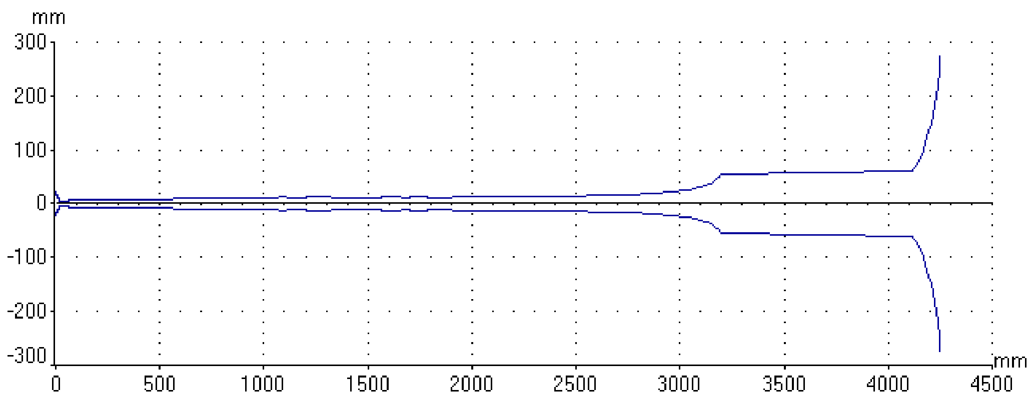


Fig. 33: Bore shape of 4671 crooked in G.

4.2 Discussion

A number of factors influencing the choice of crook for horns have been discussed. There are practical reasons for which a certain type of crooking might be preferable. While the inventionshorn is compact as it the crook is inserted in the middle of the tubing and it usually comes with only a few crooks, it also means that long crooks cannot be used as they would not fit. The intonation is distorted as the cylindrical crooks are inserted in what is otherwise a conical tube. Although the instruments measured have shown to have an acceptable harmonicity, they both suffered from rather flat resonances in its middle range. The limited range of crooks and the difficulty in tuning confirms the use of the inventionshorn as an instrument for playing solo, as indicated by Raoux' instrument called *cor solo*.

Terminal crooks provide the clear advantage that they can be exchanged as needed. This means that a new crook can be acquired to match the required tuning standard or give a better intonation; 1874 and its set of crooks provide the perfect example of different crooks in the same key used for different purposes. Inserting the crook at the mouthpiece also means that each crook can be tapered as to best approximate the ideal bore shape. Some of the measurement results of terminal crooks are excellent and show that the horns can have excellent intonation even over a range of crooks. But herein lies the disadvantage of the terminal crook, the amount of tubing to be carried with the instrument is potentially enormous and therefore impractical for transportation.

Master crook and couplers require less amount of metal as crooks and couplers are put together in combinations which achieve the required length tube length. This system can be rather impractical especially for low keys where a large number of couplers are needed; the instrument becomes unstable and wobbly while having to be played rather far away from the player's body. Unfortunately only one instrument's input impedance could be measured but it told us that a good intonation can be found in many crooking combinations. In fact, the flexibility in crooking combinations means that there can be many ways of extending the length of the instrument until its intonation is satisfactory.

Longer crooks have a large number of acoustical resonances than shorter crooks as they can 'fit more in' before the cut-off frequency. They also tend to have a darker sound as the low resonances produce oscillatory regimes. But longer instruments also have a slower response as the sound wave travels further before establishing a standing wave. Some shorter crooks have been found to be rather out of tune but they can be advantageous to play in a high register. The harmonics for the high register are more spaced out than for longer crooks where the high register's harmonics are on a diatonic or even chromatic scale; high notes can be simpler to find with a short crook. The experimental are in broad agreement with Baines' statement that medium length crooks are best built as they play the most frequently used keys.

The crooks not only affect the input impedance of an instrument but also its pitch bending potential. This means that many factors contributing to the playing experience of a horn are determined by its choice of crooking. We now understand why players would have been so reluctant to adopt the valves and renounce to crooks at the end of the 19th century. Crooks could be thought of simply being a manner of extending the amount of notes and keys played by an instrument. But even when a horn has valves to extend the range of playable in one crooking, the use of crooks determines the tuning, timbre and playability of the horn.

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